



Operational Research, Logistics & Retail

NONLINEAR MULTI-OBJECTIVE OPTIMIZATION OF A SUPPLY CHAIN UNDER FLEXIBLE CONSTRAINTS

2024/12 (v1.9)

xtractis.ai

USE CASE OBJECTIVES

GOAL Design an AI system that optimizes the logistics network between manufacturing sites and their consumer zones while maximizing Operating Income.

- PROS & BENEFITS**
- ☑ Best distribute production –as much as possible– to maximize income.
 - ☑ Systematically and quickly find the best parameters of the supply chain, considering contradictory and constrained objectives (quantities to be produced, inventory costs, transportation costs...).
 - ☑ Ultimately, simplify the organization of manufacturing sites and avoid transporting unnecessary loads and traveling over long distances.
 - ☑ Respect ecological regulations while maintaining income level.

MODEL TYPE	Regression	Multinomial Classification	Binomial Classification	Scoring	Analytical Models
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In this Use Case, we use only analytical models as they are created easily. Otherwise, we would have used fuzzy models induced by XTRACTIS® REVEAL from a reference dataset.

WORK HYPOTHESES

CHARACTERISTICS OF THE SUPPLY CHAIN The French “FUZZ Corp” uses its trucks to transport its products from its factories and warehouses to its retail areas.

It manufactures a range of 3 Products

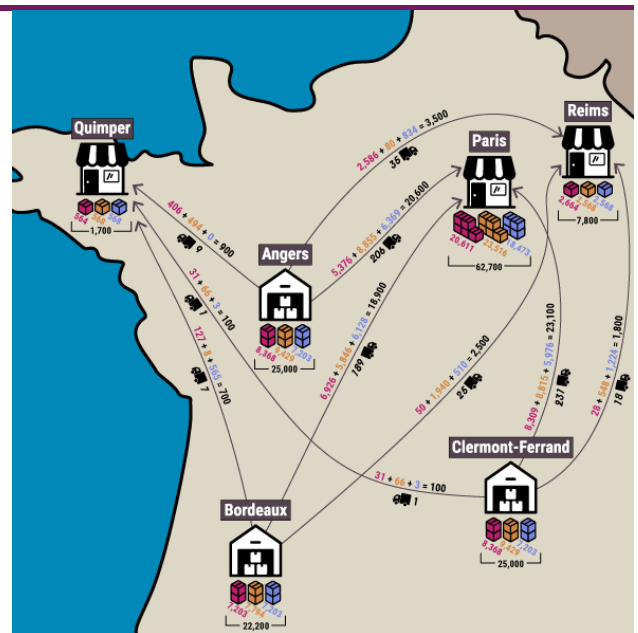
P_1 P_2 P_3

at 3 Manufacturing Sites (MS)

Angers “A”
Bordeaux “B”
Clermont-Ferrand “C”

for 3 Consumer Zones (CZ)

Paris “P”
Quimper “Q”
Reims “R”



Solution for Scenario #1

DEFINING ANALYTICAL MODELS

Formulas in appendices #1 to #5

Five realistic non-linear models put into equations the various incomes and expenses of FUZZ Corp to obtain the Operating Income of the supply chain:

- #1. **Product Manufacturing Cost:** depending on the volume of products manufactured at each MS, the unit manufacturing cost and the costs associated with the MS.
- #2. **Product Storage Cost:** depending on the storage cost of the MS, the storage cost of the CZ, the volume of products, and the costs associated with the MS.
- #3. **Product Transportation Cost:** depending on the capacity of the trucks, the quantity of products, the cost per kilometer and the distance traveled.
- #4. **Turnover:** depending on the selling price of the products, the quantity of products and a margin that varies according to the CZ.
- #5. **Operating Income:** equals to turnover minus manufacturing, storage, and transportation costs.

DEFINING REQUESTS & SCENARIOS

- 1. **Primary Objective:** *What is the maximum amount of Operating Income to reach?*
 - ▶ We obtain this amount by setting unitary prices, costs, and manufactured volumes as indicated in the appendices #1 to #4 and then by launching XTRACTIS OPTIMIZE for a single-objective optimization with no constraints to find this optimal fuzzy amount.
 - ▶ This **fuzzy objective** will be the same for all scenarios.
- 2. **Sales Objectives:** *What are the expected sales on the 3 CZs, for each of the 3 products?*
 - ▶ This means defining **9 fuzzy objectives** (intervals with approximate boundaries).
 - ▶ We model them with fuzzy intervals as described in Appendix #6.
- 3. **Supply Constraints:** *What are the production capacities of each MS, for each of the 3 products? What are the total production capacities of the 3 MSs, for all products? Are there additional constraints whether to respect investment and costs or ecological regulations?*
 - ▶ We have basically **12 binary constraints** for the 1st scenario: 9 for the production capacities of each MS, and 3 for the total production capacities of the 3 MS. For the next scenarios, we add more constraints to eventually reach **17 binary constraints** in scenarios 3 and 4.
 - ▶ We model these binary constraints with classic intervals.

The table below condenses the 4 scenarios defined for this Use Case. For each scenario, XTRACTIS OPTIMIZE will explore the multi-dimensional space of possible solutions to find the solution best satisfying each request, i.e., the best possible configuration of the supply chain. It is assumed that all objectives and constraints have the same priority.

		Scenario #1	Scenario #2	Scenario #3	Scenario #4
Primary Objective		Maximize Operating Income as much as possible while respecting Sales Objectives and all Supply Constraints			
Sales Objectives	CZ-P expected sales*	about 17,500 to 27,500	about 17,500 to 27,500	about 17,500 to 27,500	about 500 to 27,500
	CZ-Q expected sales*	about 750 to 2,250	about 750 to 2,250	about 750 to 2,250	about 500 to 2,250
	CZ-R expected sales*	about 2,750 to 4,250	about 2,750 to 4,250	about 2,750 to 4,250	about 500 to 4,250
Supply Constraints	MS-A prod. capacity*	up to 15,000	up to 15,000	up to 30,000	up to 30,000
	MS-B prod. capacity*	up to 20,000	up to 20,000	up to 20,000	up to 20,000
	MS-C prod. capacity*	up to 15,000	up to 15,000	up to 15,000	up to 15,000
	MS-A total capacity**	up to 25,000	up to 25,000	up to 50,000	up to 50,000
	MS-B total capacity**	up to 45,000	up to 45,000	up to 45,000	up to 45,000
	MS-C total capacity**	up to 25,000	up to 25,000	up to 25,000	up to 25,000
	Manufacturing Sites Limit	no	2 MSs max	2 MSs max	2 MSs max
Limitation of the ecological impact by avoiding the 2 longest routes	no	no	No transportation from MS-B to CZ-R & from MS-C to CZ-Q	No transportation from MS-B to CZ-R & from MS-C to CZ-Q	
*for each product **for all products					

XTRACTIS OPERATIONAL RESEARCH PROCESS

MULTI-OBJECTIVE FUZZY-OPTIMAL SOLUTIONS UNDER CONSTRAINTS

We have a complex non-linear process of 27 variables in total and OPTIMIZE must compute the values of each variable to maximize the Operating Income, by considering binary or fuzzy objectives and constraints depending on the scenario.

MS	CZ	Paris			Quimper			Reims		
Angers		quantity 1	quantity 2	quantity 3	quantity 4	quantity 5	quantity 6	quantity 7	quantity 8	quantity 9
Bordeaux		quantity 10	quantity 11	quantity 12	quantity 13	quantity 14	quantity 15	quantity 16	quantity 17	quantity 18
Clermont-Ferrand		quantity 19	quantity 20	quantity 21	quantity 22	quantity 23	quantity 24	quantity 25	quantity 26	quantity 27

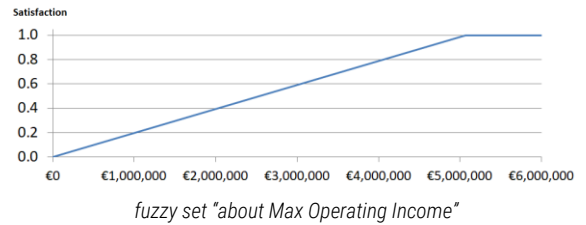
For all scenarios, XTRACTIS OPTIMIZE automatically discovers the configuration of the fuzzy-optimum supply chain for production and sales volumes, while satisfying the objectives and constraints with **the highest possible Satisfaction Degree**.

OPTIMIZATION OUTCOME

PRIMARY OBJECTIVE

First, OPTIMIZE computes that the maximum amount of Operating Income is €5,066,110.45.

So “Maximize Operating Income as much as possible” is the fuzzy objective: **Superior to about €5,066,110** for all scenarios.



Results by:



RESULTS OVERVIEW

Then, OPTIMIZE computes the 27 quantities of each product to produce at each MS, and for each CZ, to reach this objective best satisfying a global request.

A global request is a connective aggregation of elementary requests. Each elementary request is fulfilled with an individual satisfaction degree. For a binary constraint or objective, the satisfaction degree is either 0 or 1. The satisfaction degree of a fuzzy constraint or objective is a real number between 0 and 1 and the more it tends towards 1, the more the elementary request is fulfilled.

The Global Satisfaction Degree of a request is the minimum of all elementary satisfaction degrees.

The table below highlights the results which are presented in more details further on.

	Scenario #1	Scenario #2	Scenario #3	Scenario #4
Predicted Operating Income	€4,327,370.66	€2,688,952.52	€3,056,148.08	€3,851,076.65
<i>Elementary Satisfaction Degree of the Primary Objective</i>	<i>0.854</i>	<i>0.531</i>	<i>0.603</i>	<i>0.760</i>
Global Satisfaction Degree	0.854	0.530	0.603	0.760

In all scenarios, no solutions would have been found using a standard binary approach of Operational Research.

SCENARIO 1 – DISCOVERY OF THE MOST OPTIMAL SOLUTION

FUZZY REQUEST

Operating Income IS superior to about €5,066,110																																																																		
AND	<table border="1"> <tr><td>CZ-P expected sales of P₁</td><td>IS</td><td>about 17,500 to 27,500</td></tr> <tr><td>CZ-P expected sales of P₂</td><td>IS</td><td>about 17,500 to 27,500</td></tr> <tr><td>CZ-P expected sales of P₃</td><td>IS</td><td>about 17,500 to 27,500</td></tr> <tr><td>CZ-Q expected sales of P₁</td><td>IS</td><td>about 750 to 2,250</td></tr> <tr><td>CZ-Q expected sales of P₂</td><td>IS</td><td>about 750 to 2,250</td></tr> <tr><td>CZ-Q expected sales of P₃</td><td>IS</td><td>about 750 to 2,250</td></tr> <tr><td>CZ-R expected sales of P₁</td><td>IS</td><td>about 2,750 to 4,250</td></tr> <tr><td>CZ-R expected sales of P₂</td><td>IS</td><td>about 2,750 to 4,250</td></tr> <tr><td>CZ-R expected sales of P₃</td><td>IS</td><td>about 2,750 to 4,250</td></tr> </table>	CZ-P expected sales of P ₁	IS	about 17,500 to 27,500	CZ-P expected sales of P ₂	IS	about 17,500 to 27,500	CZ-P expected sales of P ₃	IS	about 17,500 to 27,500	CZ-Q expected sales of P ₁	IS	about 750 to 2,250	CZ-Q expected sales of P ₂	IS	about 750 to 2,250	CZ-Q expected sales of P ₃	IS	about 750 to 2,250	CZ-R expected sales of P ₁	IS	about 2,750 to 4,250	CZ-R expected sales of P ₂	IS	about 2,750 to 4,250	CZ-R expected sales of P ₃	IS	about 2,750 to 4,250	AND	<table border="1"> <tr><td>MS-A prod. capacity of P₁</td><td>IS</td><td>up to 15,000</td></tr> <tr><td>MS-A prod. capacity of P₂</td><td>IS</td><td>up to 15,000</td></tr> <tr><td>MS-A prod. capacity of P₃</td><td>IS</td><td>up to 15,000</td></tr> <tr><td>MS-B prod. capacity of P₁</td><td>IS</td><td>up to 20,000</td></tr> <tr><td>MS-B prod. capacity of P₂</td><td>IS</td><td>up to 20,000</td></tr> <tr><td>MS-B prod. capacity of P₃</td><td>IS</td><td>up to 20,000</td></tr> <tr><td>MS-C prod. capacity of P₁</td><td>IS</td><td>up to 15,000</td></tr> <tr><td>MS-C prod. capacity of P₂</td><td>IS</td><td>up to 15,000</td></tr> <tr><td>MS-C prod. capacity of P₃</td><td>IS</td><td>up to 15,000</td></tr> <tr><td>MS-A total capacity of P₁+P₂+P₃</td><td>IS</td><td>up to 25,000</td></tr> <tr><td>MS-B total capacity of P₁+P₂+P₃</td><td>IS</td><td>up to 45,000</td></tr> <tr><td>MS-C total capacity of P₁+P₂+P₃</td><td>IS</td><td>up to 25,000</td></tr> </table>	MS-A prod. capacity of P ₁	IS	up to 15,000	MS-A prod. capacity of P ₂	IS	up to 15,000	MS-A prod. capacity of P ₃	IS	up to 15,000	MS-B prod. capacity of P ₁	IS	up to 20,000	MS-B prod. capacity of P ₂	IS	up to 20,000	MS-B prod. capacity of P ₃	IS	up to 20,000	MS-C prod. capacity of P ₁	IS	up to 15,000	MS-C prod. capacity of P ₂	IS	up to 15,000	MS-C prod. capacity of P ₃	IS	up to 15,000	MS-A total capacity of P ₁ +P ₂ +P ₃	IS	up to 25,000	MS-B total capacity of P ₁ +P ₂ +P ₃	IS	up to 45,000	MS-C total capacity of P ₁ +P ₂ +P ₃	IS	up to 25,000
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RESULTS

Fuzzy Optimum supply chain for production and sales volumes satisfying 10 fuzzy objectives and under 12 binary constraints:

	Paris			Quimper			Reims			Productions per Product			Cumulative Productions
Angers	5,376	8,855	6,369	406	494	0	2,586	80	834	8,368	9,429	7,203	25,000
Total shipped	20,600 206	900 9	3,500 35	≤ 15,000	≤ 15,000	≤ 15,000	≤ 25,000						
Bordeaux	6,926	5,846	6,128	227	8	565	50	1,940	510	7,203	7,794	7,203	22,200
Total shipped	18,900 189	800 8	2,500 25	≤ 20,000	≤ 20,000	≤ 20,000	≤ 45,000						
Clermont-Ferrand	8,309	8,815	5,976	31	66	3	28	548	1,224	8,368	9,429	7,203	25,000
Total shipped	23,100 231	100 1	1,800 18	≤ 15,000	≤ 15,000	≤ 15,000	≤ 25,000						
Cumulative Sales	20,611	23,516	18,473	664	568	568	2,664	2,568	2,568				
Elementary Satisfaction Degree	1.000	1.000	1.000	0.931	0.854	0.854	0.931	0.854	0.854				
Elementary Satisfaction Level													

Global Satisfaction Degree



€ Operating Income

€4,327,370.66	
<i>with Elementary Satisfaction Degree of 0.854</i>	
Turnover:	€16,032,572.50
Manufacturing Cost:	€7,514,935.84
Storage Cost:	€1,628,250.00
Transport Cost:	€2,562,016.00

COMMENTS

- ▶ Each truck is filled to the maximum of its loading capacity.
- ▶ The production capacity of Angers and Clermont-Ferrand MSs limit the improvement in Operating Income because their maximum total capacity have been reached (25,000 products).
- ▶ Due to the greater demand for Paris CZ, 86.7% of the manufactured products are sold in this CZ: 82.4% of production at Angers, 85.1% of production at Bordeaux and 92.4% of production at Clermont-Ferrand are shipped to Paris.
- ▶ As the Global Satisfaction Degree ≠ 1, no solution would have been found by a standard binary approach of Operational Research. The strongest constraints are those with degrees farthest from 1.

SCENARIO 2 – DISCOVERY OF THE MOST OPTIMAL SOLUTION

FUZZY REQUEST

The Management of FUZZ Corp. decides to limit investments due to lack of capital (skills, machines, funds). Henceforth, it manufactures each product on 1 site or 2 sites at most. This second scenario adds 3 new binary constraints:

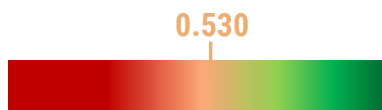
Operating Income		IS	superior to about €5,066,110				
AND	CZ-P expected sales of P ₁	IS	about 17,500 to 27,500	AND	MS-A prod. capacity of P ₁	IS	up to 15,000
	CZ-P expected sales of P ₂	IS	about 17,500 to 27,500		MS-A prod. capacity of P ₂	IS	up to 15,000
	CZ-P expected sales of P ₃	IS	about 17,500 to 27,500		MS-A prod. capacity of P ₃	IS	up to 15,000
	CZ-Q expected sales of P ₁	IS	about 750 to 2,250		MS-B prod. capacity of P ₁	IS	up to 20,000
	CZ-Q expected sales of P ₂	IS	about 750 to 2,250		MS-B prod. capacity of P ₂	IS	up to 20,000
	CZ-Q expected sales of P ₃	IS	about 750 to 2,250		MS-B prod. capacity of P ₃	IS	up to 20,000
	CZ-R expected sales of P ₁	IS	about 2,750 to 4,250		MS-C prod. capacity of P ₁	IS	up to 15,000
	CZ-R expected sales of P ₂	IS	about 2,750 to 4,250		MS-C prod. capacity of P ₂	IS	up to 15,000
	CZ-R expected sales of P ₃	IS	about 2,750 to 4,250		MS-C prod. capacity of P ₃	IS	up to 15,000
					MS-A total capacity of P ₁ +P ₂ +P ₃	IS	up to 25,000
					MS-B total capacity of P ₁ +P ₂ +P ₃	IS	up to 45,000
					MS-C total capacity of P ₁ +P ₂ +P ₃	IS	up to 25,000
					Number of sites manufacturing P ₁	IS	maximum 2
					Number of sites manufacturing P ₂	IS	maximum 2
					Number of sites manufacturing P ₃	IS	maximum 2

RESULTS

Fuzzy Optimum supply chain for production and sales volumes satisfying 10 fuzzy objectives and under 15 binary constraints:

	Paris			Quimper			Reims			Productions per Product			Cumulative Productions
Angers	12,731	0	7,769	12	0	163	2,163	0	1,426	14,906	0	9,358	24,264
Total shipped	20,500 205			175 2			3,589 36			≤ 15,000	≤ 15,000	≤ 15,000	≤ 25,000
Bordeaux	6,847	7,534	0	336	163	0	0	0	0	7,183	7,697	0	14,880
Total shipped	14,381 144			499 5			0 0			≤ 20,000	≤ 20,000	≤ 20,000	≤ 45,000
Clermont-Ferrand	0	8,841	8,557	0	0	0	0	2,163	737	0	11,004	9,294	20,298
Total shipped	17,398 174			0 0			2,900 29			≤ 15,000	≤ 15,000	≤ 15,000	≤ 25,000
Cumulative Sales	19,578	16,375	16,326	348	163	163	2,163	2,163	2,163	P ₁ is manufactured only by MS-A & MS-B			
Elementary Satisfaction Degree	1.000	0.550	0.530	0.678	0.530	0.530	0.530	0.530	0.530	P ₂ is manufactured only by MS-B & MS-C			
Elementary Satisfaction Level										P ₃ is manufactured only by MS-A & MS-C			

Global Satisfaction Degree



€ Operating Income

€2,688,952.52	
with Elementary Satisfaction Degree of 0.531	
Turnover:	€13,264,170.00
Manufacturing Cost:	€7,209,151.98
Storage Cost:	€1,338,137.50
Transport Cost:	€2,027,928.00

COMMENTS

- ▶ Since each MS now specializes in only two products, it must produce more of these products than in Scenario #1 to meet demand.
- ▶ As the unit manufacturing cost of the products increases according to the number of products manufactured for each MS, therefore the total number of manufactured products has decreased by 17.78% compared to Scenario #1.
- ▶ Due to their higher charter cost, the two longest routes in the network are unused (MS-B to CZ-R, MS-C to CZ-Q). We will add these constraints to the next scenarios, because of their ecological benefit.
- ▶ As the Global Satisfaction Degree ≠ 1, no solution would have been found by a standard binary approach of Operational Research. With 3 new constraints, the optimization problem becomes more difficult to solve.

SCENARIO 3 – DISCOVERY OF THE MOST OPTIMAL SOLUTION

FUZZY REQUEST

The 3rd scenario supposes that MS-Angers improves its productivity to double its production capacities for each product and its total capacity progresses up to 50,000 units. The Management of FUZZ Corp. also decides to prohibit any chartering on the 2 longest routes of the network. We hence add 2 new binary constraints:

Operating Income	IS superior to about €5,066,110		
AND	CZ-P expected sales of P ₁	IS about 17,500 to 27,500	AND MS-A prod. capacity of P ₁
	CZ-P expected sales of P ₂	IS about 17,500 to 27,500	IS up to 30,000
	CZ-P expected sales of P ₃	IS about 17,500 to 27,500	MS-A prod. capacity of P ₃
	CZ-Q expected sales of P ₁	IS about 750 to 2,250	IS up to 30,000
	CZ-Q expected sales of P ₂	IS about 750 to 2,250	MS-B prod. capacity of P ₁
	CZ-Q expected sales of P ₃	IS about 750 to 2,250	IS up to 20,000
	CZ-R expected sales of P ₁	IS about 2,750 to 4,250	MS-B prod. capacity of P ₂
	CZ-R expected sales of P ₂	IS about 2,750 to 4,250	IS up to 20,000
	CZ-R expected sales of P ₃	IS about 2,750 to 4,250	MS-B prod. capacity of P ₃
			IS up to 20,000
			MS-C prod. capacity of P ₁
			IS up to 15,000
			MS-C prod. capacity of P ₂
			IS up to 15,000
			MS-C prod. capacity of P ₃
			IS up to 15,000
			MS-A total capacity of P ₁ +P ₂ +P ₃
			IS up to 50,000
			MS-B total capacity of P ₁ +P ₂ +P ₃
			IS up to 45,000
			MS-C total capacity of P ₁ +P ₂ +P ₃
			IS up to 25,000
			Number of sites manufacturing P ₁
			IS maximum 2
			Number of sites manufacturing P ₂
			IS maximum 2
			Number of sites manufacturing P ₃
			IS maximum 2
			Route from MS-B to CZ-R
			IS forbidden
			Route from MS-C to CZ-Q
			IS forbidden

RESULTS

Fuzzy Optimum supply chain for production and sales volumes satisfying 10 fuzzy objectives and under 17 binary constraints:

	Paris			Quimper			Reims			Productions per Product			
													Cumulative Productions
Angers	21,537	11,712	8,251	287	254	254	619	81	1,700	22,443	12,047	10,205	44,695
Total shipped	41,500 415	795 8	2,400 24	≤ 30,000	≤ 30,000	≤ 30,000	≤ 50,000						
Bordeaux	0	0	0	0	0	0	0	0	0	0	0	0	
Total shipped	0 0	0 0	0 0	≤ 20,000	≤ 20,000	≤ 20,000	≤ 45,000						
Clermont-Ferrand	5,530	6,813	8,257	0	0	0	1,673	2,173	554	7,203	8,986	8,811	25,000
Total shipped	20,600 206	0 0	4,400 44	≤ 15,000	≤ 15,000	≤ 15,000	≤ 25,000						
Cumulative Sales	27,067	18,525	16,508	287	254	254	2,292	2,254	2,254				
Elementary Satisfaction Degree	1.000	1.000	0.603	0.630	0.603	0.603	0.634	0.603	0.603	P ₁ is manufactured only by MS-A & MS-B P ₂ is manufactured only by MS-B & MS-C P ₃ is manufactured only by MS-A & MS-C No Route from MS-B to CZ-R & MS-C to CZ-Q			
Elementary Satisfaction Level													

Global Satisfaction Degree

0.603



Operating Income

€3,056,148.08

with Elementary Satisfaction Degree of 0.603	
Turnover:	€14,473,367.50
Manufacturing Cost:	€7,890,005.42
Storage Cost:	€1,549,150.00
Transport Cost:	€1,978,064.00

COMMENTS

- ▶ MS Clermont-Ferrand limits the improvement in Operating Income because its maximum total capacity has been reached (25,000 products).
- ▶ The new capacity of MS-Angers greatly benefits the Operating Income. This MS is used at 89.4% of its new total capacity with 44,695 products manufactured. As Angers is the nearest to the CZs, the cost of transport is lower than the previous scenarios (2.46% reduction compared to Scenario #2) despite the significant increase of number of chartered trucks (17.14% increase compared to Scenario #2).
- ▶ An unexpected side effect arises with the new constraints: the uselessness of MS-Bordeaux. MS-Angers being more competitive than MS-Bordeaux, the FUZZ Corp Management could make the decision of dismantling MS-Bordeaux to transfer its skills and machines to MS-Angers.
- ▶ As the Global Satisfaction Degree ≠ 1, no solution would have been found by a standard binary approach of Operational Research.

SCENARIO 4 – DISCOVERY OF THE MOST OPTIMAL SOLUTION

FUZZY REQUEST

The previous scenarios required the delivery of a minimum volume of each product to each CZ to meet the demand. In this scenario FUZZ Corp. decides to loosen up the sales objectives and reduce this minimum number to 500 for all CZs, at the risk of generating stock shortages.

Operating Income		IS	superior to about €5,066,110		
AND	CZ-P expected sales of P ₁	IS	about 500 to 27,500	AND MS-A prod. capacity of P ₁ IS up to 30,000	
	CZ-P expected sales of P ₂	IS	about 500 to 27,500		
	CZ-P expected sales of P ₃	IS	about 500 to 27,500		
	CZ-Q expected sales of P ₁	IS	about 500 to 2,250		
	CZ-Q expected sales of P ₂	IS	about 500 to 2,250		
	CZ-Q expected sales of P ₃	IS	about 500 to 2,250		
	CZ-R expected sales of P ₁	IS	about 500 to 4,250		
	CZ-R expected sales of P ₂	IS	about 500 to 4,250		
	CZ-R expected sales of P ₃	IS	about 500 to 4,250		
					MS-A prod. capacity of P ₂ IS up to 30,000
					MS-A prod. capacity of P ₃ IS up to 30,000
					MS-B prod. capacity of P ₁ IS up to 20,000
					MS-B prod. capacity of P ₂ IS up to 20,000
					MS-B prod. capacity of P ₃ IS up to 20,000
					MS-C prod. capacity of P ₁ IS up to 15,000
					MS-C prod. capacity of P ₂ IS up to 15,000
					MS-C prod. capacity of P ₃ IS up to 15,000
				MS-A total capacity of P ₁ +P ₂ +P ₃ IS up to 50,000	
				MS-B total capacity of P ₁ +P ₂ +P ₃ IS up to 45,000	
				MS-C total capacity of P ₁ +P ₂ +P ₃ IS up to 25,000	
				Number of sites manufacturing P ₁ IS maximum 2	
				Number of sites manufacturing P ₂ IS maximum 2	
				Number of sites manufacturing P ₃ IS maximum 2	
				Route from MS-B to CZ-R IS forbidden	
				Route from MS-C to CZ-Q IS forbidden	

RESULTS

Fuzzy Optimum supply chain for production and sales volumes satisfying 10 modified fuzzy objectives and under 17 binary constraints:

	Paris			Quimper			Reims			Productions per Product			Cumulative Productions
	P ₁	P ₂	P ₃	P ₁	P ₂	P ₃	P ₁	P ₂	P ₃	P ₁	P ₂	P ₃	
Angers	19,894	11,834	6,972	289	200	211	104	176	20	20,287	12,210	7,203	39,700
Total shipped	38,700 387	700 7	300 3	≤ 30,000	≤ 30,000	≤ 30,000	≤ 50,000						
Bordeaux	0	0	0	0	0	0	0	0	0	0	0	0	0
Total shipped	0 0	0 0	0 0	≤ 20,000	≤ 20,000	≤ 20,000	≤ 45,000						
Clermont-Ferrand	8,206	9,471	7,023	0	0	0	96	24	180	8,302	9,495	7,203	25,000
Total shipped	24,700 247	0 0	300 3	≤ 15,000	≤ 15,000	≤ 15,000	≤ 25,000						
Cumulative Sales	28,100	21,305	13,995	289	200	211	200	200	200				
Elementary Satisfaction Degree	0.760	1.000	1.000	0.831	0.760	0.769	0.760	0.760	0.760	P ₁ is manufactured only by MS-A & MS-B P ₂ is manufactured only by MS-B & MS-C P ₃ is manufactured only by MS-A & MS-C No Route from MS-B to CZ-R & MS-C to CZ-Q			
Elementary Satisfaction Level													

Global Satisfaction Degree

0.760



Operating Income

€3,851,076.65

with Elementary Satisfaction Degree of 0.760

Turnover:	€12,723,065.00
Manufacturing Cost:	€5,626,088.35
Storage Cost:	€1,452,500.00
Transport Cost:	€1,793,400.00

COMMENTS

- ▶ MS Clermont-Ferrand limits the improvement in Operating Income because its maximum total capacity has been reached (25,000 products).
- ▶ Each truck is filled to the maximum of its loading capacity.
- ▶ Products for Reims decreased by 91.18% compared to Scenario #3: supplying this CZ is to the detriment of the Operating Income. CZ-Paris reduces its supply of P₃ product (-15.22%), in favor of P₁ (+3.82%) and P₂ (+15.01%) products, which are more profitable.
- ▶ The result is a significant reduction in costs and expenses for a smaller reduction in turnover. This allows a strong increase in Operational Income of 26.01% compared to Scenario #3 and the achievement of the highest Global Satisfaction Degree since Scenario #1, which had far fewer constraints.
- ▶ MS-Bordeaux is still dismantled in favor of the MS-Angers.
- ▶ As the Global Satisfaction Degree ≠ 1, no solution would have been found by a standard binary approach of Operational Research.

APPENDICES

1 – Manufacturing Cost

For each Manufacturing Site (MS), the unit manufacturing cost of products P_i increases according to the number of products manufactured: this symbolizes the scarcity of raw materials necessary for manufacturing and therefore the increase in their cost of obtaining.

→ The manufacturing cost of a P_i product for an MS is not linear according to the number of parts designed on this same MS.

At a constant volume of P_i products manufactured on a given MS, the manufacturing costs depend on the P_i product designed:

- P_3 costs 5 times more to manufacture than P_2
- P_2 costs 5 times more to manufacture than P_1

A 12.5% increase is also applied to products manufactured in Bordeaux because this MS, close to a major metropolis, generates higher charges (rent, taxes, etc.).

$$\text{ManufacturingCost}(\text{MS}) = k_{\text{MS}} \times \left(\begin{array}{l} 10 \times \max \left(P_1(\text{MS}); \left(\frac{P_1(\text{MS})}{45} \right)^{1.75} \right) \\ + 50 \times \max \left(P_2(\text{MS}); \left(\frac{P_2(\text{MS})}{45} \right)^{1.75} \right) \\ + 250 \times \max \left(P_3(\text{MS}); \left(\frac{P_3(\text{MS})}{45} \right)^{1.75} \right) \end{array} \right)$$

$\text{MS} \in \{A, B, C\}$: Manufacturing Sites

$k_{\text{MS}} = 1.125$ if $\text{MS} = B$

$k_{\text{MS}} = 1$ else

$P_i(\text{MS}) \in \{P_1(\text{MS}), P_2(\text{MS}), P_3(\text{MS})\}$: number of products made on the MS

$$\text{ManufacturingCost} = \sum_{\text{MS}} \text{ManufacturingCost}(\text{MS})$$

$\text{MS} \in \{A, B, C\}$: Manufacturing Sites

Application with the fuzzy-optimal solution of Scenario #1

Bordeaux MS manufactures 22,200 products distributed as follows:

- 7,203 P_1 (Paris: 6,926, Quimper: 227, Reims: 50)
- 7,794 P_2 (Paris: 5,846, Quimper: 8, Reims: 1,940)
- 7,203 P_3 (Paris: 6,128, Quimper: 565, Reims: 510)

$$\text{ManufacturingCost}(B) = 1.125 \times \left(\begin{array}{l} 10 \times \max \left(7,203; \left(\frac{7,203}{45} \right)^{1.75} \right) \\ + 50 \times \max \left(7,794; \left(\frac{7,794}{45} \right)^{1.75} \right) \\ + 250 \times \max \left(7,203; \left(\frac{7,203}{45} \right)^{1.75} \right) \end{array} \right)$$

$$\text{ManufacturingCost}(B) = \text{€}2,572,078.83$$

2 – Storage Cost

Each manufactured P_i product is stored twice. The first time, it is stored close to its Manufacturing Site (MS), pending shipment. The second time, it is stored close to its Consumer Zone (CZ), pending its sale. The cost of storing a product therefore depends on its MS, its CZ and its volume (to facilitate the overall understanding of the problem and the formulas, the volume of each product is arbitrarily set at 1 m^3). The unit cost of storing a product is 10 €/m^3 .

Like manufacturing costs (increased in Bordeaux), storage cost increases of 12.5% and 25% are respectively applied to products stored in Bordeaux and Paris (11.25 €/m^3 and 12.5 €/m^3).

$$\text{StorageCost}(\text{MS}) = k_{\text{MS}} \times 10 \times \sum_i (P_i(\text{MS}))$$

$\text{MS} \in \{A, B, C\}$: Manufacturing Sites

$k_{\text{MS}} = 1.125$ if $\text{MS} = B$

$k_{\text{MS}} = 1$ else

$P_i(\text{MS}) \in \{P_1(\text{MS}), P_2(\text{MS}), P_3(\text{MS})\}$: number of products stored on the MS

$$\text{StorageCost}(\text{CZ}) = k_{\text{CZ}} \times 10 \times \sum_i (P_i(\text{CZ}))$$

$\text{CZ} \in \{P, Q, R\}$: Consumer Zones

$k_{\text{CZ}} = 1.25$ if $\text{CZ} = P$

$k_{\text{CZ}} = 1$ else

$P_i(\text{CZ}) \in \{P_1(\text{CZ}), P_2(\text{CZ}), P_3(\text{CZ})\}$: number of products stored on the CZ

$$\text{StorageCost} = \sum_{\text{MS}} \text{StorageCost}(\text{MS}) + \sum_{\text{CZ}} \text{StorageCost}(\text{CZ})$$

$\text{MS} \in \{A, B, C\}$: Manufacturing Sites

$\text{CZ} \in \{P, Q, R\}$: Consumer Zones

Application with the fuzzy-optimal solution of Scenario #1

Bordeaux MS stores 22,200 products distributed as follows:

- 7,203 P_1 (Paris: 6,926; Quimper: 227; Reims: 50)
- 7,794 P_2 (Paris: 5,846; Quimper: 8; Reims: 1,940)
- 7,203 P_3 (Paris: 6,128; Quimper: 565; Reims: 510)

$$\text{StorageCost}(B) = 1.125 \times 10 \times (7,203 + 7,794 + 7,203) = \text{€}249,750.00$$

CZ-Paris stores 62,600 products distributed as follows:

- 20,611 P_1 (Angers: 5,376; Bordeaux: 6,926; Clermont-Ferrand: 8,309)
- 23,516 P_2 (Angers: 8,855; Bordeaux: 5,846; Clermont-Ferrand: 8,815)
- 18,473 P_3 (Angers: 6,369; Bordeaux: 6,128; Clermont-Ferrand: 5,976)

$$\text{StorageCost}(P) = 1.25 \times 10 \times (20,611 + 23,516 + 18,473) = \text{€}782,500.00$$

3 – Transport Cost

The company charters trucks with a volume capacity of 100 m^3 and the cost per kilometer is 8 €/km . As each product P_i occupies 1 m^3 , each truck can transport 100 products, regardless of the distribution of the different products.

The following table defines each distance between an $\text{MS} \in \{A, B, C\}$ and a $\text{CZ} \in \{P, Q, R\}$:

	CZ-P: Paris	CZ-Q: Quimper	CZ-R: Reims
MS-A: Angers	296 km	312 km	430 km
MS-B: Bordeaux	584 km	577 km	718 km
MS-C: Clermont-Ferrand	423 km	755 km	556 km

$$\text{TransportCost}(\text{MS}, \text{CZ}) = \text{distance}(\text{MS}, \text{CZ}) \times 8 \times \text{ceiling} \left(\frac{\sum_i P_i(\text{MS}, \text{CZ})}{100} \right)$$

$\text{MS} \in \{A, B, C\}$: Manufacturing Sites

$\text{CZ} \in \{P, Q, R\}$: Consumer Zones

$\text{distance}(\text{MS}, \text{CZ})$: number of kilometers between the MS and the CZ

$\text{ceiling}(x)$: returns the smallest integer greater than or equal to x

$P_i(\text{MS}, \text{CZ})$: total number of P_i products manufactured on the MS and intended for the CZ

$$\text{TransportCost} = \sum_{MS} \sum_{CZ} \text{TransportCost}(MS, CZ)$$

MS ∈ {A, B, C}: Manufacturing Sites

CZ ∈ {P, Q, R}: Consumer Zones

Application with the fuzzy-optimal solution of Scenario #1

MS-Bordeaux manufactures 22,200 products distributed as follows:

- 7,203 P₁ (Paris: 6,926, Quimper: 227, Reims: 50)
- 7,794 P₂ (Paris: 5,846, Quimper: 8, Reims: 1,940)
- 7,203 P₃ (Paris: 6,128, Quimper: 565, Reims: 510)

That is:

- 18,900 products for Paris (P₁: 6,926; P₂: 5,846; P₃: 6,128)
- 800 products for Quimper (P₁: 227; P₂: 8; P₃: 565)
- 2,500 products for Reims (P₁: 50; P₂: 1,940; P₃: 510)

$$\text{TransportCost}(B, P) = 584 \times 8 \times \text{ceiling}\left(\frac{6,926 + 5,846 + 6,128}{100}\right) = \text{€}883,008.00$$

$$\text{TransportCost}(B, Q) = 577 \times 8 \times \text{ceiling}\left(\frac{227 + 8 + 565}{100}\right) = \text{€}36,928.00$$

$$\text{TransportCost}(B, R) = 718 \times 8 \times \text{ceiling}\left(\frac{50 + 1,940 + 510}{100}\right) = \text{€}143,600.00$$

4 - Turnover

Each product has a selling price depending on its level of quality:

- P₁ costs €70
- P₂ costs €140
- P₃ costs €360

Like the increased manufacturing costs in Bordeaux, the selling price of products sold in Paris is increased by 25%.

$$\text{Turnover}(CZ) = k_{CZ} \times (70 \times P_1(CZ) + 140 \times P_2(CZ) + 360 \times P_3(CZ))$$

CZ ∈ {P, Q, R}: Consumer Zones

k_{CZ} = 1.25 if CZ = P

k_{CZ} = 1 else

P_i(CZ): total number of P_i products in destination to CZ

$$\text{Turnover} = \sum_{CZ} \text{Turnover}(CZ)$$

CZ ∈ {P, Q, R}: Consumer Zones

Application with the fuzzy-optimal solution of Scenario #1

Paris CZ sells 62,700 products distributed as follows:

- 20,611 P₁ (Angers: 5,376; Bordeaux: 6,926; Clermont-Ferrand: 8,309)
- 23,516 P₂ (Angers: 8,855; Bordeaux: 5,846; Clermont-Ferrand: 8,815)
- 18,473 P₃ (Angers: 6,369; Bordeaux: 6,128; Clermont-Ferrand: 5,976)

$$\begin{aligned} \text{Turnover}(P) &= 1.25 \times (70 \times 20,611 + 140 \times 23,516 + 360 \times 18,473) \\ \text{Turnover}(P) &= \text{€}14,231,612.50 \end{aligned}$$

5 - Operating Income

The Operating Income is obtained by subtracting the sum of the various costs and charges from the turnover. This is the variable that symbolizes the main objective of the project: to maximize the Operating Income.

$$\text{OperatingIncome} = \text{Turnover} - \text{ManufacturingCost} - \text{StorageCost} - \text{TransportCost}$$

Application with the fuzzy-optimal solution of Scenario #1

$$\begin{aligned} &16,032,572.50 - 7,514,935.84 - 1,628,250 - 2,562,016 \\ &= \text{€}4,327,370.66 \end{aligned}$$

6 - Definition of Fuzzy Intervals

The fuzzy request in scenarios 1, 2 and 3 for the sales objectives is:

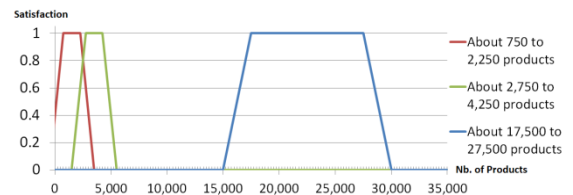
- CZ-Paris: *about 17,500 to 27,500* for each product
- CZ-Quimper: *about 750 to 2,250* for each product
- CZ-Reims: *about 2,750 to 4,250* for each product

It is modeled by the following fuzzy intervals:

~ *750 to 2,250* = (-500 750 2,250 3,500) products

~ *2,750 to 4,250* = (1,500 2,750 4,250 5,500) products

~ *17,500 to 27,500* = (15,000 17,500 27,500 30,000) products



Remark: We accept that no products will be delivered to CZ-Quimper (Satisfaction Degree = 0.4).

The fuzzy request in scenario 4 for the sales objectives is:

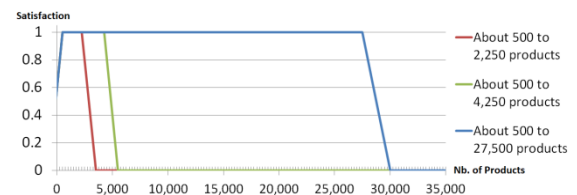
- CZ-Paris: *about 500 to 27,500* for each product
- CZ-Quimper: *about 500 to 2,250* for each product
- CZ-Reims: *about 500 to 4,250* for each product

It is modeled by the following fuzzy intervals:

~ *500 to 2,250* = (-750 500 2,250 3,500) products

~ *500 to 4,250* = (-750 500 4,250 5,500) products

~ *500 to 27,500* = (-750 500 27,500 30,000) products



Remark: Satisfaction Degree is limited to 0.6 if no product is delivered to one of the 3 CZs, which represents a higher tolerance to possible stockouts.