Operational Research / Retail / Logistics

NONLINEAR MULTI-OBJECTIVE OPTIMIZATION OF A SUPPLY CHAIN UNDER FLEXIBLE CONSTRAINTS

2023/09 (v1.8)

xtractis.ai

USE CASE OBJECTIVES

GOAL

Design an AI system that optimizes the logistics network between manufacturing sites and their consumer zones while maximizing Operating Income.

PROS & BENEFITS

- ☑ Best distribute production —as much as possible— to maximize income.
- ☑ Systematically and quickly find the best parameters of the supply chain, considering contradictory and constrained objectives (quantities to be produced, inventory costs, transportation costs…).
- ☑ Ultimately, simplify the organization of manufacturing sites and avoid transporting unnecessary loads and traveling over long distances.
- ☑ Respect ecological regulations while maintaining income level.

MODEL TYPE

Regression | Multinomial Classification | Binomial Classification | Scoring | **Analytical Models**

In this Use Case, we use only analytical models as they are created easily. Otherwise, we would have used fuzzy models induced by XTRACTIS® REVEAL from a reference dataset.

WORK HYPOTHESES

CHARACTERISTICS OF THE SUPPLY CHAIN

The French “FUZZ Corp” uses its trucks to transport its products from its factories and warehouses to its retail areas.

It manufactures a range of 3 Products

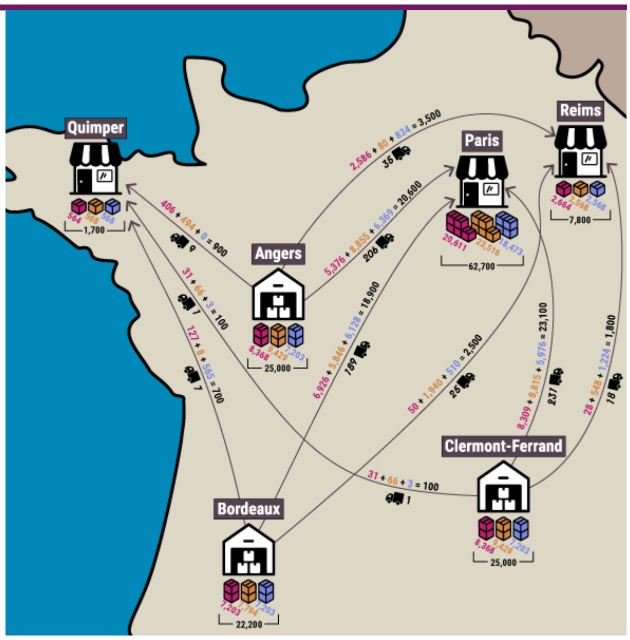
P_1 P_2 P_3

at 3 Manufacturing Sites (MS)

- Angers “A”
- Bordeaux “B”
- Clermont-Ferrand “C”

for 3 Consumer Zones (CZ)

- Paris “P”
- Quimper “Q”
- Reims “R”



Solution for Scenario #1

DEFINING ANALYTICAL MODELS

Formulas in appendices #1 to #5

Five realistic non-linear models put into equations the various incomes and expenses of FUZZ Corp to obtain the Operating Income of the supply chain:

#1. **Product Manufacturing Cost**: depending on the volume of products manufactured at each MS, the unit manufacturing cost and the costs associated with the MS.

#2. **Product Storage Cost**: depending on the storage cost of the MS, the storage cost of the CZ, the volume of products, and the costs associated with the MS.

#3. **Product Transportation Cost**: depending on the capacity of the trucks, the quantity of products, the cost per kilometer and the distance traveled.

#4. **Turnover**: depending on the selling price of the products, the quantity of products and a margin that varies according to the CZ.

#5. **Operating Income**: equals to turnover minus manufacturing, storage, and transportation costs.

DEFINING REQUESTS & SCENARIOS

1. **Primary Objective**: *What is the maximum amount of Operating Income to reach?*
   - We obtain this amount by setting unitary prices, costs, and manufactured volumes as indicated in the appendices #1 to #4 and then by launching XTRACTIS OPTIMIZE for a single-objective optimization with no constraints to find this optimal fuzzy amount.
   - This **fuzzy objective** will be the same for all scenarios.

2. **Sales Objectives**: *What are the expected sales on the 3 CZs, for each of the 3 products?*
   - This means defining **9 fuzzy objectives** (intervals with approximate boundaries).
   - We model them with fuzzy intervals as described in Appendix #6.

3. **Supply Constraints**: *What are the production capacities of each MS, for each of the 3 products? What are the total production capacities of the 3 MSs, for all products? Are there additional constraints whether to respect investment and costs or ecological regulations?*
   - We have basically **12 binary constraints** for the 1st scenario: 9 for the production capacities of each MS, and 3 for the total production capacities of the 3 MS. For the next scenarios, we add more constraints to eventually reach **17 binary constraints** in scenarios 3 and 4.
   - We model these binary constraints with classic intervals.

The table below condenses the 4 scenarios defined for this Use Case. For each scenario, XTRACTIS OPTIMIZE will explore the multi-dimensional space of possible solutions to find the solution best satisfying each request, i.e., the best possible configuration of the supply chain. It is assumed that all objectives and constraints have the same priority.

		Scenario #1	Scenario #2	Scenario #3	Scenario #4
Primary Objective		Maximize Operating Income as much as possible while respecting Sales Objectives and all Supply Constraints			
Sales Objectives	CZ-P expected sales*	about 17,500 to 27,500	about 17,500 to 27,500	about 17,500 to 27,500	about **500** to 27,500
	CZ-Q expected sales*	about 750 to 2,250	about 750 to 2,250	about 750 to 2,250	about **500** to 2,250
	CZ-R expected sales*	about 2,750 to 4,250	about 2,750 to 4,250	about 2,750 to 4,250	about **500** to 4,250
Supply Constraints	MS-A prod. capacity*	up to 15,000	up to 15,000	up to **30,000**	up to 30,000
	MS-B prod. capacity*	up to 20,000	up to 20,000	up to 20,000	up to 20,000
	MS-C prod. capacity*	up to 15,000	up to 15,000	up to 15,000	up to 15,000
	MS-A total capacity**	up to 25,000	up to 25,000	up to **50,000**	up to 50,000
	MS-B total capacity**	up to 45,000	up to 45,000	up to 45,000	up to 45,000
	MS-C total capacity**	up to 25,000	up to 25,000	up to 25,000	up to 25,000
	Manufacturing Sites Limit	no	**2 MSs max**	2 MSs max	2 MSs max
*for **each** product **for **all** products	Limitation of the ecological impact by avoiding the 2 longest routes	no	no	**No transportation from MS-B to CZ-R & from MS-C to CZ-Q**	No transportation from MS-B to CZ-R & from MS-C to CZ-Q

XTRACTIS OPERATIONAL RESEARCH PROCESS

MULTI-OBJECTIVE FUZZY-OPTIMAL SOLUTIONS UNDER CONSTRAINTS

We have a complex non-linear process of 27 variables in total and OPTIMIZE must compute the values of each variable to maximize the Operating Income, by considering binary or fuzzy objectives and constraints depending on the scenario.

MS \ CZ	Paris			Quimper			Reims		
Angers	quantity 1	quantity 2	quantity 3	quantity 4	quantity 5	quantity 6	quantity 7	quantity 8	quantity 9
Bordeaux	quantity 10	quantity 11	quantity 12	quantity 13	quantity 14	quantity 15	quantity 16	quantity 17	quantity 18
Clermont-Ferrand	quantity 19	quantity 20	quantity 21	quantity 22	quantity 23	quantity 24	quantity 25	quantity 26	quantity 27

For all scenarios, XTRACTIS OPTIMIZE automatically discovers the configuration of the fuzzy-optimum supply chain for production and sales volumes, while satisfying the objectives and constraints with **the highest possible Satisfaction Degree**.

OPTIMIZATION OUTCOME

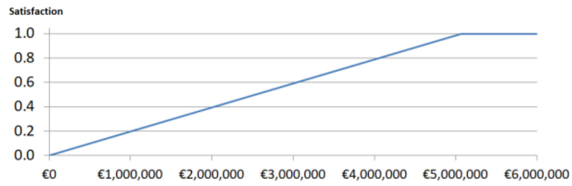
Results by:

XTRACTIS® OPTIMIZE v13.0.46103

PRIMARY OBJECTIVE

First, OPTIMIZE computes that the maximum amount of Operating Income is €5,066,110.45.

So "Maximize Operating Income as much as possible" is the fuzzy objective: Superior to about €5,066,110 for all scenarios.



fuzzy set "about Max Operating Income"

RESULTS OVERVIEW

Then, OPTIMIZE computes the 27 quantities of each product to produce at each MS, and for each CZ, to reach this objective best satisfying a global request.

A global request is a connective aggregation of elementary requests. Each elementary request is fulfilled with an individual satisfaction degree. For a binary constraint or objective, the satisfaction degree is either 0 or 1. The satisfaction degree of a fuzzy constraint or objective is a real number between 0 and 1 and the more it tends towards 1, the more the elementary request is fulfilled.

The Global Satisfaction Degree of a request is the minimum of all elementary satisfaction degrees.

The table below highlights the results which are presented in more details further on.

	Scenario #1	Scenario #2	Scenario #3	Scenario #4
Predicted Operating Income *Elementary Satisfaction Degree of the Primary Objective*	**€4,327,370.66** *0.854*	**€2,688,952.52** *0.531*	**€3,056,148.08** *0.603*	**€3,851,076.65** *0.760*
Global Satisfaction Degree	**0.854**	**0.530**	**0.603**	**0.760**

In all scenarios, no solutions would have been found using a standard binary approach of Operational Research.

SCENARIO 1 — DISCOVERY OF THE MOST OPTIMAL SOLUTION

Results by: XTRACTIS® OPTIMIZE v.13.0.46103

FUZZY REQUEST

Operating Income IS superior to about €5,066,110

AND	CZ-P expected sales of P_1	IS	about 17,500 to 27,500
	CZ-P expected sales of P_2	IS	about 17,500 to 27,500
	CZ-P expected sales of P_3	IS	about 17,500 to 27,500
	CZ-Q expected sales of P_1	IS	about 750 to 2,250
	CZ-Q expected sales of P_2	IS	about 750 to 2,250
	CZ-Q expected sales of P_3	IS	about 750 to 2,250
	CZ-R expected sales of P_1	IS	about 2,750 to 4,250
	CZ-R expected sales of P_2	IS	about 2,750 to 4,250
	CZ-R expected sales of P_3	IS	about 2,750 to 4,250

AND	MS-A prod. capacity of P_1	IS	up to 15,000
	MS-A prod. capacity of P_2	IS	up to 15,000
	MS-A prod. capacity of P_3	IS	up to 15,000
	MS-B prod. capacity of P_1	IS	up to 20,000
	MS-B prod. capacity of P_2	IS	up to 20,000
	MS-B prod. capacity of P_3	IS	up to 20,000
	MS-C prod. capacity of P_1	IS	up to 15,000
	MS-C prod. capacity of P_2	IS	up to 15,000
	MS-C prod. capacity of P_3	IS	up to 15,000
	MS-A total capacity of $P_1+P_2+P_3$	IS	up to 25,000
	MS-B total capacity of $P_1+P_2+P_3$	IS	up to 45,000
	MS-C total capacity of $P_1+P_2+P_3$	IS	up to 25,000

RESULTS

Fuzzy Optimum supply chain for production and sales volumes satisfying **10 fuzzy objectives** and under **12 binary constraints**:

	Paris			Quimper			Reims			Productions per Product			Cumulative Productions
	P_1	P_2	P_3	P_1	P_2	P_3	P_1	P_2	P_3	P_1	P_2	P_3	
Angers	5,376	8,855	6,369	406	494	0	2,586	80	834	8,368 ≤ 15,000 ✓	9,429 ≤ 15,000 ✓	7,203 ≤ 15,000 ✓	25,000 STOP ≤ 25,000 ✓
Total shipped	20,600		206	900		9	3,500		35				
Bordeaux	6,926	5,846	6,128	227	8	565	50	1,940	510	7,203 ≤ 20,000 ✓	7,794 ≤ 20,000 ✓	7,203 ≤ 20,000 ✓	22,200 ≤ 45,000 ✓
Total shipped	18,900		189	800		8	2,500		25				
Clermont-Ferrand	8,309	8,815	5,976	31	66	3	28	548	1,224	8,368 ≤ 15,000 ✓	9,429 ≤ 15,000 ✓	7,203 ≤ 15,000 ✓	25,000 STOP ≤ 25,000 ✓
Total shipped	23,100		231	100		1	1,800		18				
Cumulative Sales	20,611	23,516	18,473	664	568	568	2,664	2,568	2,568				
Elementary Satisfaction Degree	1.000	1.000	1.000	0.931	0.854	0.854	0.931	0.854	0.854				
Elementary Satisfaction Level													

Global Satisfaction Degree

0.854

Operating Income

€4,327,370.66 *with Elementary Satisfaction Degree of 0.854*	
Turnover:	€16,032,572.50
Manufacturing Cost:	€7,514,935.84
Storage Cost:	€1,628,250.00
Transport Cost:	€2,562,016.00

COMMENTS

- Each truck is filled to the maximum of its loading capacity.
- The production capacity of Angers and Clermont-Ferrand MSs limit the improvement in Operating Income because their maximum total capacity have been reached (25,000 products).
- Due to the greater demand for Paris CZ, 86.7% of the manufactured products are sold in this CZ: 82.4% of production at Angers, 85.1% of production at Bordeaux and 92.4% of production at Clermont-Ferrand are shipped to Paris.
- As the Global Satisfaction Degree ≠ 1, no solution would have been found by a standard binary approach of Operational Research. The strongest constraints are those with degrees farthest from 1.

SCENARIO 2 — DISCOVERY OF THE MOST OPTIMAL SOLUTION

Results by: XTRACTIS® OPTIMIZE v.13.0.46103

FUZZY REQUEST

The Management of FUZZ Corp. decides to limit investments due to lack of capital (skills, machines, funds). Henceforth, it manufactures each product on 1 site or 2 sites at most. This second scenario adds 3 new binary constraints:

	Operating Income	IS	superior to about €5,066,110
AND	CZ-P expected sales of P_1	IS	about 17,500 to 27,500
	CZ-P expected sales of P_2	IS	about 17,500 to 27,500
	CZ-P expected sales of P_3	IS	about 17,500 to 27,500
	CZ-Q expected sales of P_1	IS	about 750 to 2,250
	CZ-Q expected sales of P_2	IS	about 750 to 2,250
	CZ-Q expected sales of P_3	IS	about 750 to 2,250
	CZ-R expected sales of P_1	IS	about 2,750 to 4,250
	CZ-R expected sales of P_2	IS	about 2,750 to 4,250
	CZ-R expected sales of P_3	IS	about 2,750 to 4,250

AND	MS-A prod. capacity of P_1	IS	up to 15,000
	MS-A prod. capacity of P_2	IS	up to 15,000
	MS-A prod. capacity of P_3	IS	up to 15,000
	MS-B prod. capacity of P_1	IS	up to 20,000
	MS-B prod. capacity of P_2	IS	up to 20,000
	MS-B prod. capacity of P_3	IS	up to 20,000
	MS-C prod. capacity of P_1	IS	up to 15,000
	MS-C prod. capacity of P_2	IS	up to 15,000
	MS-C prod. capacity of P_3	IS	up to 15,000
	MS-A total capacity of $P_1+P_2+P_3$	IS	up to 25,000
	MS-B total capacity of $P_1+P_2+P_3$	IS	up to 45,000
	MS-C total capacity of $P_1+P_2+P_3$	IS	up to 25,000
	Number of sites manufacturing P_1	**IS**	**maximum 2**
	Number of sites manufacturing P_2	**IS**	**maximum 2**
	Number of sites manufacturing P_3	**IS**	**maximum 2**

RESULTS

Fuzzy Optimum supply chain for production and sales volumes satisfying **10 fuzzy objectives** and under **15 binary constraints**:

	Paris P_1	Paris P_2	Paris P_3	Quimper P_1	Quimper P_2	Quimper P_3	Reims P_1	Reims P_2	Reims P_3	Productions per Product P_1	Productions per Product P_2	Productions per Product P_3	Cumulative Productions
Angers	12,731	0	7,769	12	0	163	2,163	0	1,426	14,906 ≤ 15,000 ✓	0 ≤ 15,000 ✓	9,358 ≤ 15,000 ✓	24,264 ≤ 25,000 ✓
Total shipped	20,500		205	175		2	3,589		36				
Bordeaux	6,847	7,534	0	336	163	0	0	0	0	7,183 ≤ 20,000 ✓	7,697 ≤ 20,000 ✓	0 ≤ 20,000 ✓	14,880 ≤ 45,000 ✓
Total shipped	14,381		144	499		5	0		0				
Clermont-Ferrand	0	8,841	8,557	0	0	0	0	2,163	737	0 ≤ 15,000 ✓	11,004 ≤ 15,000 ✓	9,294 ≤ 15,000 ✓	20,298 ≤ 25,000 ✓
Total shipped	17,398		174	0		0	2,900		29				
Cumulative Sales	19,578	16,375	16,326	348	163	163	2,163	2,163	2,163				
Elementary Satisfaction Degree	1.000	0.550	0.530	0.678	0.530	0.530	0.530	0.530	0.530				
Elementary Satisfaction Level													

P_1 is manufactured only by MS-A & MS-B ✓
P_2 is manufactured only by MS-B & MS-C ✓
P_3 is manufactured only by MS-A & MS-C ✓

Global Satisfaction Degree

0.530

€ Operating Income

€2,688,952.52	
with Elementary Satisfaction Degree of 0.531	
Turnover:	€13,264,170.00
Manufacturing Cost:	€7,209,151.98
Storage Cost:	€1,338,137.50
Transport Cost:	€2,027,928.00

COMMENTS

- Since each MS now specializes in only two products, it must produce more of these products than in Scenario #1 to meet demand.
- As the unit manufacturing cost of the products increases according to the number of products manufactured for each MS, therefore the total number of manufactured products has decreased by 17.78% compared to Scenario #1.
- Due to their higher charter cost, the two longest routes in the network are unused (MS-B to CZ-R, MS-C to CZ-Q). We will add these constraints to the next scenarios, because of their ecological benefit.
- As the Global Satisfaction Degree ≠ 1, no solution would have been found by a standard binary approach of Operational Research. With 3 new constraints, the optimization problem becomes more difficult to solve.

SCENARIO 3 — DISCOVERY OF THE MOST OPTIMAL SOLUTION

Results by: XTRACTIS® OPTIMIZE v.13.0.46103

FUZZY REQUEST

The 3rd scenario supposes that MS-Angers improves its productivity to double its production capacities for each product and its total capacity progresses up to 50,000 units. The Management of FUZZ Corp. also decides to prohibit any chartering on the 2 longest routes of the network. We hence add 2 new binary constraints:

Operating Income IS superior to about €5,066,110

AND	CZ-P expected sales of P_1	IS	about 17,500 to 27,500
	CZ-P expected sales of P_2	IS	about 17,500 to 27,500
	CZ-P expected sales of P_3	IS	about 17,500 to 27,500
	CZ-Q expected sales of P_1	IS	about 750 to 2,250
	CZ-Q expected sales of P_2	IS	about 750 to 2,250
	CZ-Q expected sales of P_3	IS	about 750 to 2,250
	CZ-R expected sales of P_1	IS	about 2,750 to 4,250
	CZ-R expected sales of P_2	IS	about 2,750 to 4,250
	CZ-R expected sales of P_3	IS	about 2,750 to 4,250

AND	MS-A prod. capacity of P_1	IS	up to **30,000**
	MS-A prod. capacity of P_2	IS	up to **30,000**
	MS-A prod. capacity of P_3	IS	up to **30,000**
	MS-B prod. capacity of P_1	IS	up to 20,000
	MS-B prod. capacity of P_2	IS	up to 20,000
	MS-B prod. capacity of P_3	IS	up to 20,000
	MS-C prod. capacity of P_1	IS	up to 15,000
	MS-C prod. capacity of P_2	IS	up to 15,000
	MS-C prod. capacity of P_3	IS	up to 15,000
	MS-A total capacity of $P_1+P_2+P_3$	IS	up to **50,000**
	MS-B total capacity of $P_1+P_2+P_3$	IS	up to 45,000
	MS-C total capacity of $P_1+P_2+P_3$	IS	up to 25,000
	Number of sites manufacturing P_1	IS	maximum 2
	Number of sites manufacturing P_2	IS	maximum 2
	Number of sites manufacturing P_3	IS	maximum 2
	Route from MS-B to CZ-R	IS	**forbidden**
	Route from MS-C to CZ-Q	IS	**forbidden**

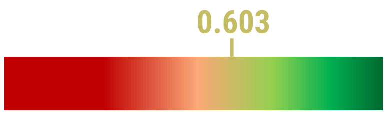
RESULTS

Fuzzy Optimum supply chain for production and sales volumes satisfying **10 fuzzy objectives** and under **17 binary constraints**:

	Paris P_1	Paris P_2	Paris P_3	Quimper P_1	Quimper P_2	Quimper P_3	Reims P_1	Reims P_2	Reims P_3	Productions per Product P_1	Productions per Product P_2	Productions per Product P_3	Cumulative Productions
Angers	21,537	11,712	8,251	287	254	254	619	81	1,700	22,443 ≤ 30,000 ✓	12,047 ≤ 30,000 ✓	10,205 ≤ 30,000 ✓	44,695 ≤ 50,000 ✓
Total shipped	41,500		415	795		8	2,400		24				
Bordeaux	0	0	0	0	0	0	0	0	0	0 ≤ 20,000 ✓	0 ≤ 20,000 ✓	0 ≤ 20,000 ✓	0 ≤ 45,000 ✓
Total shipped	0		0	0		0	0		0				
Clermont-Ferrand	5,530	6,813	8,257	0	0	0	1,673	2,173	554	7,203 ≤ 15,000 ✓	8,986 ≤ 15,000 ✓	8,811 ≤ 15,000 ✓	25,000 STOP ≤ 25,000 ✓
Total shipped	20,600		206	0		0	4,400		44				
Cumulative Sales	27,067	18,525	16,508	287	254	254	2,292	2,254	2,254				
Elementary Satisfaction Degree	1.000	1.000	0.603	0.630	0.603	0.603	0.634	0.603	0.603				
Elementary Satisfaction Level													

P_1 is manufactured only by MS-A & MS-B ✓
P_2 is manufactured only by MS-B & MS-C ✓
P_3 is manufactured only by MS-A & MS-C ✓
No Route from MS-B to CZ-R & MS-C to CZ-Q ✓ ✓

Global Satisfaction Degree



Operating Income

€3,056,148.08	
with Elementary Satisfaction Degree of 0.603	
Turnover:	€14,473,367.50
Manufacturing Cost:	€7,890,005.42
Storage Cost:	€1,549,150.00
Transport Cost:	€1,978,064.00

COMMENTS

- MS Clermont-Ferrand limits the improvement in Operating Income because its maximum total capacity has been reached (25,000 products).
- The new capacity of MS-Angers greatly benefits the Operating Income. This MS is used at 89.4% of its new total capacity with 44,695 products manufactured. As Angers is the nearest to the CZs, the cost of transport is lower than the previous scenarios (2.46% reduction compared to Scenario #2) despite the significant increase of number of chartered trucks (17.14% increase compared to Scenario #2).
- An unexpected side effect arises with the new constraints: the uselessness of MS-Bordeaux. MS-Angers being more competitive than MS-Bordeaux, the FUZZ Corp Management could make the decision of dismantling MS-Bordeaux to transfer its skills and machines to MS-Angers.
- As the Global Satisfaction Degree ≠ 1, no solution would have been found by a standard binary approach of Operational Research.

SCENARIO 4 — DISCOVERY OF THE MOST OPTIMAL SOLUTION

Results by: XTRACTIS® OPTIMIZE v.13.0.46103

FUZZY REQUEST

The previous scenarios required the delivery of a minimum volume of each product to each CZ to meet the demand. In this scenario FUZZ Corp. decides to loosen up the sales objectives and reduce this minimum number to 500 for all CZs, at the risk of generating stock shortages.

	Operating Income	IS	superior to about €5,066,110
AND	CZ-P expected sales of P_1	IS	about **500** to 27,500
	CZ-P expected sales of P_2	IS	about **500** to 27,500
	CZ-P expected sales of P_3	IS	about **500** to 27,500
	CZ-Q expected sales of P_1	IS	about **500** to 2,250
	CZ-Q expected sales of P_2	IS	about **500** to 2,250
	CZ-Q expected sales of P_3	IS	about **500** to 2,250
	CZ-R expected sales of P_1	IS	about **500** to 4,250
	CZ-R expected sales of P_2	IS	about **500** to 4,250
	CZ-R expected sales of P_3	IS	about **500** to 4,250

AND	MS-A prod. capacity of P_1	IS	up to 30,000
	MS-A prod. capacity of P_2	IS	up to 30,000
	MS-A prod. capacity of P_3	IS	up to 30,000
	MS-B prod. capacity of P_1	IS	up to 20,000
	MS-B prod. capacity of P_2	IS	up to 20,000
	MS-B prod. capacity of P_3	IS	up to 20,000
	MS-C prod. capacity of P_1	IS	up to 15,000
	MS-C prod. capacity of P_2	IS	up to 15,000
	MS-C prod. capacity of P_3	IS	up to 15,000
	MS-A total capacity of $P_1+P_2+P_3$	IS	up to 50,000
	MS-B total capacity of $P_1+P_2+P_3$	IS	up to 45,000
	MS-C total capacity of $P_1+P_2+P_3$	IS	up to 25,000
	Number of sites manufacturing P_1	IS	maximum 2
	Number of sites manufacturing P_2	IS	maximum 2
	Number of sites manufacturing P_3	IS	maximum 2
	Route from MS-B to CZ-R	IS	forbidden
	Route from MS-C to CZ-Q	IS	forbidden

RESULTS

Fuzzy Optimum supply chain for production and sales volumes satisfying **10 modified fuzzy objectives** and under **17 binary constraints**:

	Paris P_1	Paris P_2	Paris P_3	Quimper P_1	Quimper P_2	Quimper P_3	Reims P_1	Reims P_2	Reims P_3	Productions per Product P_1	Productions per Product P_2	Productions per Product P_3	Cumulative Productions
Angers	19,894	11,834	6,972	289	200	211	104	176	20	20,287 ≤ 30,000 ✓	12,210 ≤ 30,000 ✓	7,203 ≤ 30,000 ✓	39,700 ≤ 50,000 ✓
Total shipped	38,700		387	700		7	300		3				
Bordeaux	0	0	0	0	0	0	0	0	0	0 ≤ 20,000 ✓	0 ≤ 20,000 ✓	0 ≤ 20,000 ✓	0 ≤ 45,000 ✓
Total shipped	0		0	0		0	0		0				
Clermont-Ferrand	8,206	9,471	7,023	0	0	0	96	24	180	8,302 ≤ 15,000 ✓	9,495 ≤ 15,000 ✓	7,203 ≤ 15,000 ✓	25,000 STOP ≤ 25,000 ✓
Total shipped	24,700		247	0		0	300		3				
Cumulative Sales	28,100	21,305	13,995	289	200	211	200	200	200				
Elementary Satisfaction Degree	0.760	1.000	1.000	0.831	0.760	0.769	0.760	0.760	0.760				
Elementary Satisfaction Level													

P_1 is manufactured only by MS-A & MS-B ✓
P_2 is manufactured only by MS-B & MS-C ✓
P_3 is manufactured only by MS-A & MS-C ✓
No Route from MS-B to CZ-R & MS-C to CZ-Q ✓ ✓

Global Satisfaction Degree

0.760

Operating Income

€3,851,076.65 *with Elementary Satisfaction Degree of 0.760*	
Turnover:	€12,723,065.00
Manufacturing Cost:	€5,626,088.35
Storage Cost:	€1,452,500.00
Transport Cost:	€1,793,400.00

COMMENTS

- MS Clermont-Ferrand limits the improvement in Operating Income because its maximum total capacity has been reached (25,000 products).
- Each truck is filled to the maximum of its loading capacity.
- Products for Reims decreased by 91.18% compared to Scenario #3: supplying this CZ is to the detriment of the Operating Income. CZ-Paris reduces its supply of P_3 product (-15.22%), in favor of P_1 (+3.82%) and P_2 (+15.01%) products, which are more profitable.
- The result is a significant reduction in costs and expenses for a smaller reduction in turnover. This allows a strong increase in Operational Income of 26.01% compared to Scenario #3 and the achievement of the highest Global Satisfaction Degree since Scenario #1, which had far fewer constraints.
- MS-Bordeaux is still dismantled in favor of the MS-Angers.
- As the Global Satisfaction Degree ≠ 1, no solution would have been found by a standard binary approach of Operational Research.

APPENDICES

1 - Manufacturing Cost

For each Manufacturing Site (MS), the unit manufacturing cost of products P_i increases according to the number of products manufactured: this symbolizes the scarcity of raw materials necessary for manufacturing and therefore the increase in their cost of obtaining.

→ The manufacturing cost of a P_i product for an MS is not linear according to the number of parts designed on this same MS.

At a constant volume of P_i products manufactured on a given MS, the manufacturing costs depend on the P_i product designed:

- P_3 costs 5 times more to manufacture than P_2
- P_2 costs 5 times more to manufacture than P_1

A 12.5% increase is also applied to products manufactured in Bordeaux because this MS, close to a major metropolis, generates higher charges (rent, taxes, etc.).

$$\mathrm{ManufacturingCost(MS)} = k_{MS}\times\begin{pmatrix} 10\times\max\left(P_1(MS);\left(\frac{P_1(MS)}{45}\right)^{1.75}\right) \\ +50\times\max\left(P_2(MS);\left(\frac{P_2(MS)}{45}\right)^{1.75}\right) \\ +250\times\max\left(P_3(MS);\left(\frac{P_3(MS)}{45}\right)^{1.75}\right)\end{pmatrix}$$

$MS \in \{A, B, C\}$: Manufacturing Sites
$k_{MS} = 1.125$ if MS = B
$k_{MS} = 1$ else
$P_i(MS) \in \{P_1(MS), P_2(MS), P_3(MS)\}$: number of products made on the MS

$$\boldsymbol{ManufacturingCost} = \sum_{MS}\mathrm{ManufacturingCost(MS)}$$

$MS \in \{A, B, C\}$: Manufacturing Sites

Application with the fuzzy-optimal solution of Scenario #1

Bordeaux MS manufactures 22,200 products distributed as follows:

- 7,203 P_1 (Paris: 6,926, Quimper: 227, Reims: 50)
- 7,794 P_2 (Paris: 5,846, Quimper: 8, Reims: 1,940)
- 7,203 P_3 (Paris: 6,128, Quimper: 565, Reims: 510)

$$\mathrm{ManufacturingCost(B)} = 1.125\times\begin{pmatrix} 10\times\max\left(7{,}203;\left(\frac{7{,}203}{45}\right)^{1.75}\right) \\ +50\times\max\left(7{,}794;\left(\frac{7{,}794}{45}\right)^{1.75}\right) \\ +250\times\max\left(7{,}203;\left(\frac{7{,}203}{45}\right)^{1.75}\right)\end{pmatrix}$$

$$\mathrm{ManufactoringCost(B)} = €2{,}572{,}078.83$$

2 - Storage Cost

Each manufactured P_i product is stored twice. The first time, it is stored close to its Manufacturing Site (MS), pending shipment. The second time, it is stored close to its Consumer Zone (CZ), pending its sale. The cost of storing a product therefore depends on its MS, its CZ and its volume (to facilitate the overall understanding of the problem and the formulas, the volume of each product is arbitrarily set at 1 m³). The unit cost of storing a product is 10€/m³.

Like manufacturing costs (increased in Bordeaux), storage cost increases of 12.5% and 25% are respectively applied to products stored in Bordeaux and Paris (11.25€/m³ and 12.5€/m³).

$$\mathrm{StorageCost(MS)} = k_{MS}\times 10\times\sum_i\left(P_i(MS)\right)$$

$MS \in \{A, B, C\}$: Manufacturing Sites
$k_{MS} = 1.125$ if MS = B
$k_{MS} = 1$ else
$P_i(MS) \in \{P_1(MS), P_2(MS), P_3(MS)\}$: number of products stored on the MS

$$\mathrm{StorageCost(CZ)} = k_{CZ}\times 10\times\sum_i\left(P_i(CZ)\right)$$

$CZ \in \{P, Q, R\}$: Consumer Zones
$k_{CZ} = 1.25$ if CZ = P
$k_{CZ} = 1$ else
$P_i(CZ) \in \{P_1(CZ), P_2(CZ), P_3(CZ)\}$: number of products stored on the CZ

$$\boldsymbol{StorageCost} = \sum_{MS}\mathrm{StorageCost(MS)} + \sum_{CZ}\mathrm{StorageCost(CZ)}$$

$MS \in \{A, B, C\}$: Manufacturing Sites
$CZ \in \{P, Q, R\}$: Consumer Zones

Application with the fuzzy-optimal solution of Scenario #1

Bordeaux MS stores 22,200 products distributed as follows:

- 7,203 P_1 (Paris: 6,926; Quimper: 227; Reims: 50)
- 7,794 P_2 (Paris: 5,846; Quimper: 8; Reims: 1,940)
- 7,203 P_3 (Paris: 6,128; Quimper: 565; Reims: 510)

$$\mathrm{StorageCost(B)} = 1.125\times10\times(7{,}203 + 7{,}794 + 7{,}203) = €249{,}750.00$$

CZ-Paris stores 62,600 products distributed as follows:

- 20,611 P_1 (Angers: 5,376; Bordeaux: 6,926; Clermont-Ferrand: 8,309)
- 23,516 P_2 (Angers: 8,855; Bordeaux: 5,846; Clermont-Ferrand: 8,815)
- 18,473 P_3 (Angers: 6,369; Bordeaux: 6,128; Clermont-Ferrand: 5,976)

$$\mathrm{StorageCost(P)} = 1.25\times10\times(20{,}611 + 23{,}516 + 18{,}473) = €782{,}500.00$$

3 - Transport Cost

The company charters trucks with a volume capacity of 100 m³ and the cost per kilometer is 8€/km. As each product P_i occupies 1 m³, each truck can transport 100 products, regardless of the distribution of the different products.

The following table defines each distance between an MS ∈ {A, B, C} and a CZ ∈ {P, Q, R}:

	CZ-P: Paris	CZ-Q: Quimper	CZ-R: Reims
MS-A: Angers	296 km	312 km	430 km
MS-B: Bordeaux	584 km	577 km	718 km
MS-C: Clermont-Ferrand	423 km	755 km	556 km

$$\mathrm{TransportCost(MS, CZ)} = \mathrm{distance(MS, CZ)}\times 8\times\mathrm{ceiling}\left(\frac{\sum_i P_i(MS, CZ)}{100}\right)$$

$MS \in \{A, B, C\}$: Manufacturing Sites
$CZ \in \{P, Q, R\}$: Consumer Zones
distance(MS,CZ): number of kilometers between the MS and the CZ
ceiling(x): returns the smallest integer greater than or equal to x
$P_i(MS,CZ)$: total number of P_i products manufactured on the MS and intended for the CZ

$$\textbf{TransportCost} = \sum_{MS}\sum_{CZ} \text{TransportCost}(\text{MS}, \text{CZ})$$

$\text{MS} \in \{\text{A}, \text{B}, \text{C}\}$: Manufacturing Sites
$\text{CZ} \in \{\text{P}, \text{Q}, \text{R}\}$: Consumer Zones

Application with the fuzzy-optimal solution of Scenario #1

MS-Bordeaux manufactures 22,200 products distributed as follows:

- 7,203 P_1 (Paris: 6,926, Quimper: 227, Reims: 50)
- 7,794 P_2 (Paris: 5,846, Quimper: 8, Reims: 1,940)
- 7,203 P_3 (Paris: 6,128, Quimper: 565, Reims: 510)

That is:

- 18,900 products for Paris (P_1: 6,926; P_2: 5,846; P_3: 6,128)
- 800 products for Quimper (P_1: 227; P_2: 8; P_3: 565)
- 2,500 products for Reims (P_1: 50; P_2: 1,940; P_3: 510)

$$\text{TransportCost}(\text{B}, \text{P}) = 584 \times 8 \times \text{ceiling}\left(\frac{6{,}926 + 5{,}846 + 6{,}128}{100}\right) = €883{,}008.00$$

$$\text{TransportCost}(\text{B}, \text{Q}) = 577 \times 8 \times \text{ceiling}\left(\frac{227 + 8 + 565}{100}\right) = €36{,}928.00$$

$$\text{TransportCost}(\text{B}, \text{R}) = 718 \times 8 \times \text{ceiling}\left(\frac{50 + 1{,}940 + 510}{100}\right) = €143{,}600.00$$

4 – Turnover

Each product has a selling price depending on its level of quality:

- P_1 costs €70
- P_2 costs €140
- P_3 costs €360

Like the increased manufacturing costs in Bordeaux, the selling price of products sold in Paris is increased by 25%.

$$\text{Turnover(CZ)} = k_{CZ} \times \left(70 \times P_1(\text{CZ}) + 140 \times P_2(\text{CZ}) + 360 \times P_3(\text{CZ})\right)$$

$\text{CZ} \in \{\text{P}, \text{Q}, \text{R}\}$: Consumer Zones
k_{CZ} = 1.25 if CZ = P
k_{CZ} = 1 else
$P_i(\text{CZ})$: total number of P_i products in destination to CZ

$$\textbf{Turnover} = \sum_{CZ} \text{Turnover(CZ)}$$

$\text{CZ} \in \{\text{P}, \text{Q}, \text{R}\}$: Consumer Zones

Application with the fuzzy-optimal solution of Scenario #1

Paris CZ sells 62,700 products distributed as follows:

- 20,611 P_1 (Angers: 5,376; Bordeaux: 6,926; Clermont-Ferrand: 8,309)
- 23,516 P_2 (Angers: 8,855; Bordeaux: 5,846; Clermont-Ferrand: 8,815)
- 18,473 P_3 (Angers: 6,369; Bordeaux: 6,128; Clermont-Ferrand: 5,976)

$$\text{Turnover(P)} = 1.25 \times (70 \times 20{,}611 + 140 \times 23{,}516 + 360 \times 18{,}473)$$
$$\text{Turnover(P)} = €14{,}231{,}612.50$$

5 - Operating Income

The Operating Income is obtained by subtracting the sum of the various costs and charges from the turnover. This is the variable that symbolizes the main objective of the project: to maximize the Operating Income.

$$\textbf{OperatingIncome} = \text{Turnover} - \text{ManufacturingCost} - \text{StorageCost} - \text{TransportCost}$$

Application with the fuzzy-optimal solution of Scenario #1

$$16{,}032{,}572.50 - 7{,}514{,}935.84 - 1{,}628{,}250 - 2{,}562{,}016 = \mathbf{4{,}327{,}370.66}$$

6 - Definition of Fuzzy Intervals

The fuzzy request in scenarios 1, 2 and 3 for the sales objectives is:

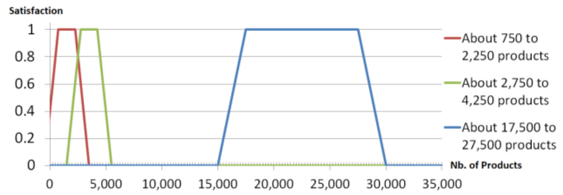
CZ-Paris: *about 17,500 to 27,500* for each product
CZ-Quimper: *about 750 to 2,250* for each product
CZ-Reims: *about 2,750 to 4,250* for each product

It is modeled by the following fuzzy intervals:

~ 750 to 2,250 = (-500 750 2,250 3,500) products

~ 2,750 to 4,250 = (1,500 2,750 4,250 5,500) products

~ 17,500 to 27,500 = (15,000 17,500 27,500 30,000) products



Remark: We accept that no products will be delivered to CZ-Quimper (Satisfaction Degree = 0.4).

The fuzzy request in scenario 4 for the sales objectives is:

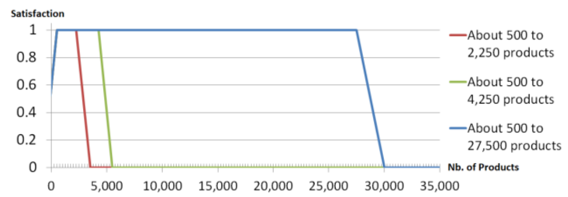
CZ-Paris: *about 500 to 27,500* for each product
CZ-Quimper: *about 500 to 2,250* for each product
CZ-Reims: *about 500 to 4,250* for each product

It is modeled by the following fuzzy intervals:

~ 500 to 2,250 = (-750 500 2,250 3,500) products

~ 500 to 4,250 = (-750 500 4,250 5,500) products

~ 500 to 27,500 = (-750 500 27,500 30,000) products



Remark: Satisfaction Degree is limited to 0.6 if no product is delivered to one of the 3 CZs, which represents a higher tolerance to possible stockouts.