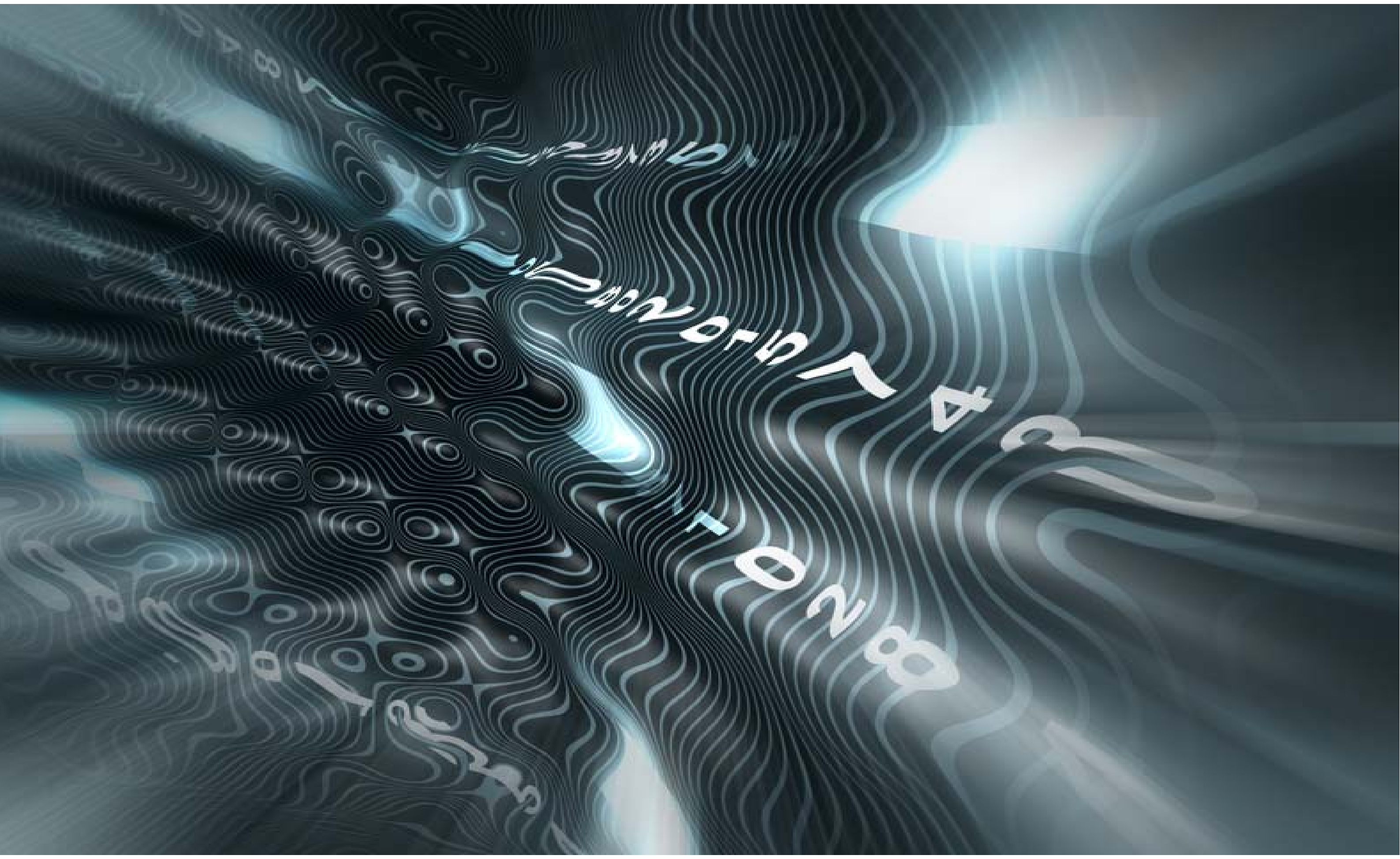
White Paper

Fuzzy Logic and Fuzzy Inference Systems

Introduction and Properties



Author: Prof. Zyed ZALILA
zyed.zalila@intellitech.fr

First publication: 12/2006
Revision: v2.4 – 12/2013

INTELLITECH – 14 rue du Fonds Pernant – Technopolis III – 60200 Compiègne, FRANCE
Tel: +33 (0)3 44 23 48 – Web: www.intellitech.ai – www.xtractis.ai
A Private Limited Company with 206 400 Euros capital – R.C.S. Compiègne B 419 510 128 – SIRET 419 510 128 00020 – Code APE 731Z

Warning

The entirety of this document is protected by copyright. Reproduction rights are reserved. Quotations from any part of the document must necessarily include the following reference:

Zalila, Z. (2006-2013) *Fuzzy Logic and Fuzzy Inference Systems: Introduction and Properties,* White Paper, v2.4, intellitech, Compiègne, France, December 2013, 12 p, http://xtractis.ai/en/a-unique-mathematical-and-algorithmic-approach/

Contents

1. FUZZY THEORY, FUZZY LOGIC

Fuzzy mathematics, also known generically as fuzzy theory, includes a number of theories which are generalizations or extensions of their classic equivalents: thus the theory of fuzzy sets is an extension of set theory, fuzzy logic is an extension of binary logic, the theory of fuzzy quantities is an extension of number and interval theory, possibility theory extends probability theory and more generally the theory of fuzzy measure extends the measure theory. All these theories offer formally rigorous concepts, techniques and methods for collecting, representing and analysing fuzzy data. The specificity of fuzzy data is that it is imprecise, uncertain and subjective and these three main fuzzy characteristics often co-exist. A whole range of other synonyms fall under this notion of fuzziness, such as knowledge which is poorly specified, poorly described, imperfect, vague, qualitative, linguistic, partial, incomplete or approximate.

This conceptual leap makes mathematics more useful and greatly increases potential applications, because these notions provide a closer approximation to the real world. Fuzzy mathematics also offers the possibility to select and apply ad hoc operators depending on both the problem to be resolved and the personality of the decision-maker (optimistic, pessimistic or compromising behaviour…).

Because fuzzy logic is nuanced and gradual, it more closely approaches human logic allowing the measure of possibility to become an accurate replacement for the measure of probability, when the available information is sparse and/or of poor quality. This is particularly the case in sensory assessment panels, where only a small number of individuals act as sensors, who are very often imprecise. Formally, fuzzy theory defines an interface between qualitative/symbolic and quantitative/numeric concepts. From a practical point of view, it offers a natural approach to the resolution of multidimensional and complex problems characterized by strong interactions of the components involved, where Human is both a sensor and a decision-maker/actuator.

FUZZY SETS, FUZZY PARTITION, FUZZY RULE

Fuzzy theory came into being in the 30's with the development of multivalent logics by the Polish School of Logic; this work was led by Lukasiewicz (Lukasiewicz 1920, 1930). At the same time, the philosopher and logician Bertrand Russell pointed out that classical logic was poorly adapted for the formalization of natural language, which includes many vague and poorly specified terms (Russel 1923). However, in 1937, Max Black proposed formalizing vague predicates, commonly used in natural language, by the innovative concept of numerical membership function (Black 1937). It only remained for Lotfi Zadeh in 1965, to put together all these premises to result in the formal definition of fuzzy logic, a nuanced and gradual logic which he used to try to mimic human logic as closely as possible.

Introduced by Zadeh (Zadeh 1965), the fuzzy set concept allows vague classes with poorly defined frontiers to be represented, such as "old person", "high temperature", "sharp bend", "concentrated solution"… Starting from a universal set, which is usually numeric, such as the description of an age in years, a temperature in degrees, a curve radius measured in metres or a concentration measured in moles per litre, Zadeh defines linguistic concepts by means of a functional application which associates every element in the universal set with a degree of membership. In this way, a fuzzy set allows a natural interface to be created between a numeric/quantitative value and a symbolic/qualitative value.

BINARITY = DISCONTINUITY AT THE BOUNDARIES

In classic set theory, there are only two possible states of membership: either element x belongs to set A, or it does not belong to the set. In a similar way, this binarity of states also exists in classical logic: a proposal can be either true or false; if it is true, the opposite is necessarily false and vice versa. In probability theory, an event occurs or does not occur. Whilst the binary approach is necessary in formal mathematical demonstrations, it becomes restrictive when resolving problems of the real world.

Let us remember that a classic set A can be defined by a characteristic function μ_A, which allocates the number 1 to all the elements which are members of that set, and the number 0 to all elements which do not belong to that set. For example, for an airline company, the set of young travellers will include all the travellers less than or equal to 25 years of age (Figure 1).

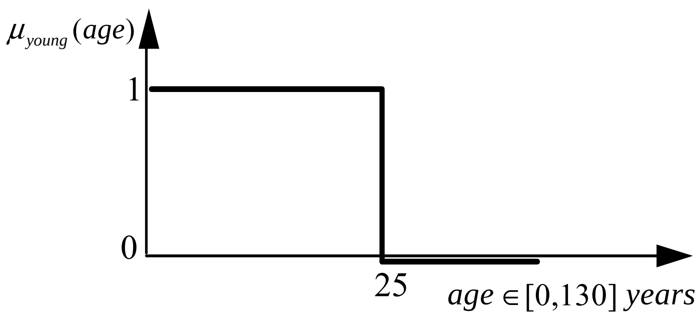


Figure 1 - Crisp definition of "young traveller"

Inevitably a threshold effect emerges: a young traveller merely has to have his/her birthday the day before his/her trip, to no longer benefit from the cheap fares for which he/she would have been eligible if he/she had taken his/her plane just one day before. In a single day this young person has changed "airline" category from "young traveller" to that of a "not young traveller". This discontinuous transition is specific to classic sets and Aristotelian logic where there is no intermediate level between 0 and 1, between belonging and not belonging, between True and False, between Black and White. And yet in everyday life we intuitively feel that the passage from "youth" to "adult" occurs in a far less drastic and more gradual way.

FUZZY = SMOOTH BOUNDARIES

Fuzziness enables the fundamental notion of graduated belonging to be introduced (the degree of membership can take any value between 0 and 1). This allows certain elements to belong more or less to any given set; this set is then qualified as fuzzy. For example for the border police the category "young traveller" is better defined by the fuzzy set shown in Figure 2.

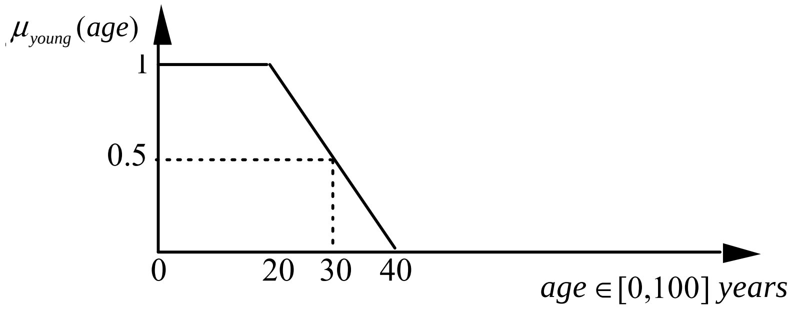


Figure 2 - Fuzzy definition of "young traveller"

This model enables people of less than 20 years old to be considered as totally young (degree = 1): it defines the prototypes of this fuzzy class; a 30 year old traveller is considered more or less young (degree = 0.5). However, a person over the age of 40 no longer belongs to this "young" set (degree = 0). Other vague categories such as "a fortyish traveller" or "a senior traveller" can also be specified so that any traveller crossing the frontier to be qualified (Figure 3).

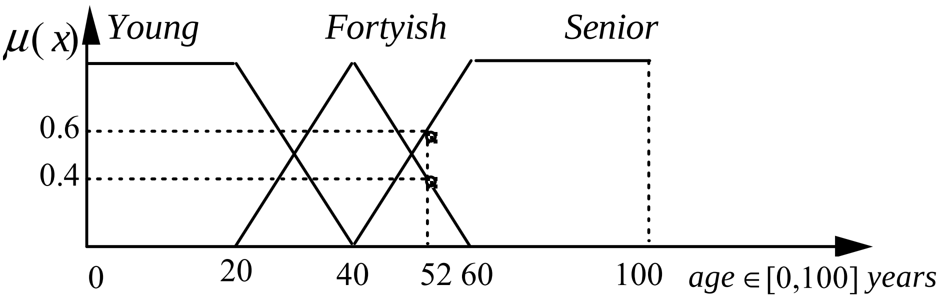


Figure 3 - fuzzy partition with three classes: "young" "fortyish", "senior"

The overlap of the fuzzy subsets means that a 52-year old traveller can be considered as somewhat senior (degree = 0.6), but also being a bit further from the "fortyish" traveller set (degree = 0.4). This overlap of classes is the basis of the robustness of fuzzy systems: it allows a progressive change of state and consequently a progressive convergence towards the final decision. Thus, the nuanced membership within the fuzzy set concept allows the definition of gradual linguistic categories in a similar way to the processing mechanism of the human brain.

This conceptual leap, which at first sight could seem insignificant, in fact overturns certain founding principles of classical mathematics. Firstly, in fuzzy theory there is an infinite number, not a unique definition, of operators for modelling the AND (conjunction, intersection), the OR (disjunction, union) the NOT (negation, complementation) and the

IF….THEN (implication, deduction). In addition, established laws such as the law of excluded middle[1] and the law of non-contradiction[2] are generally no longer respected.

From fuzzy variables qualified by fuzzy classes, it is possible to construct decision fuzzy rules:

IF (the traveller is **Young** **OR** **Fortyish**) **AND** (the traveller is returning from an **Exotic** destination) **AND** (the trip was **Brief**) **THEN** Proceed with a **Thorough** search of the traveller and his luggage using the police dog patrol.

A fuzzy system is composed of a compilation of fuzzy rules. By means of an inference engine implementing approximate reasoning schemas derived from the Generalized Modus Ponens (Zadeh 1975) (Zalila 1993), appropriate fuzzy rules can be fired to calculate the final decision of the fuzzy system.

2. FUZZY MODELLING

The objective of modelling is to obtain a formal model which describes a natural, human or industrial process to understand it better (e.g. ocean sedimentation processes or the strategy for evaluation of a product by one consumer segment), and/or with a view to predicting the effects of the process (e.g. the acceptability of a new product by this consumer segment, the efficiency and the toxicity of a new molecule, the early cancer diagnosis for a new patient).

Based on the concept of fuzzy rules[3], the fuzzy modelling approach is far more intuitive than classic modelling approaches, whether probabilistic or analytical based on differential equation or partial derivative systems. Fuzzy rules are effectively the basic building blocks for elementary expert knowledge modelling, where fuzzy inputs are connected non-linearly with either crisp or fuzzy outputs. The standard fuzzy approach requires a priori knowledge of the process, which is then directly modelled using fuzzy rules (Zadeh 1968, 1973) (Mamdani 1974) (Mamdani & Assilian 1975). However, there are a number of situations, in particular in sensory analysis, where either there is no heuristic expertise on the process being studied or there is only an implicit knowledge i.e. which cannot be made explicit by the expert[4]. In this case, the only available information is the reference input-output data. From these reference bases, x**tractis**® GENERATE will automatically induce the set of fuzzy rules which best represents the process under study.

Fuzzy systems are universal approximators of non-linear functions connecting output variables to observed variables (Kosko 1992) (Wang & Mendel 1992). In theory they can model any complex non-linear dynamic process or system with a very large number of

---

[1] The Law of Excluded Middle states that for any proposition *P*, either *P* is true or its negation is true [(P or not P) is true].
[2] The Law of Non Contradiction states that a proposition *P* and its negation cannot be true at the same time [(P and not P) is false]
[3] According to the model "If *Premise*…Then *Conclusion*" derived from cognitive psychology. Such rules are intrinsically fuzzy because they use nuanced concepts in both the *Premise* and *Conclusion* sections.
[4] Heuristics are generally embedded in the subconscious (Gigerenzer & al. 1999).

variables, which can be highly interactive (natural phenomena, physico-chemical processes, behaviour of a human operator…).

All fuzzy models benefit from the four important properties of fuzzy systems (Zalila, 1993-2013):

INTERPRETABILITY

Defining a descriptive and qualitative model of a process, for example the behaviour of a decision-making agent, becomes relatively straightforward because it is mainly based on the use of linguistic rules which describe the agent's expertise, heuristic strategies and know-how. Fuzzy rules are a very useful way of connecting the decision-maker's actions to the description of the environmental conditions encountered during the decision-making process, even when this description is either partial or vague.

The architectural simplicity of the model means that programming is also simpler and more flexible with lower maintenance costs. In addition, in the context of a multidisciplinary design project for any technological system involving strong interaction with an operator, this formal set of rules has a number of advantages. Indeed, the clarity with which it can be read and the similarity with natural language, facilitate dialogue between scientists and engineers, ergonomists and cognitive psychologists. The engineer no longer needs to provide a qualitative explanation for something that has been quantitatively modelled using differential or partial derivative equations.

LOCALITY

Each rule models the behaviour of the process under study in a given situation. This behaviour can be described imprecisely, uncertainly or subjectively. Any modification in the behaviour of the process in a given situation is obtained by modifying local rules, specialized in the description of this state of the world. Defining fuzzy rules comes to defining multidimensional fuzzy cells that map the decision space[5] (Figure 4). As the rules are fuzzy, the respective cells overlap.

[5] One fuzzy cell = one fuzzy rule

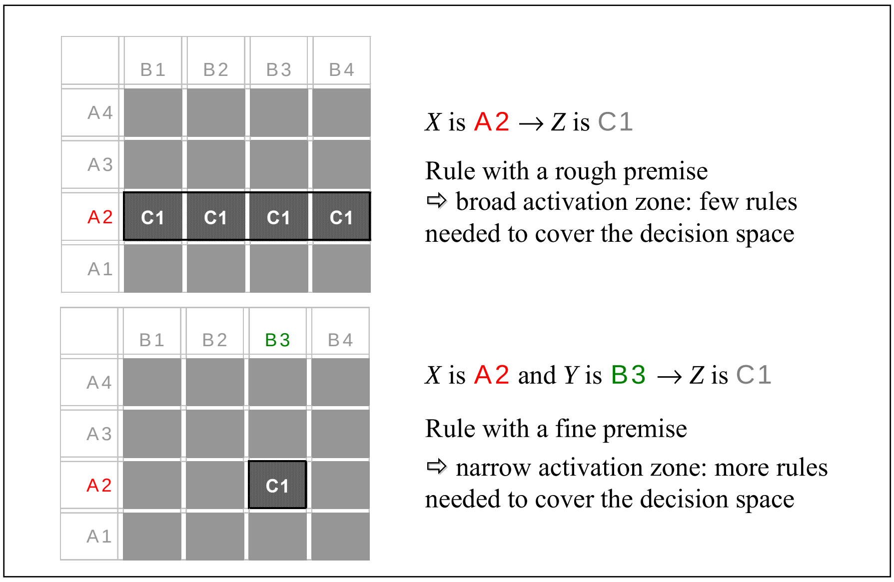


Figure 4 - Locality of fuzzy rules

This fuzzy cellular meshing of the decision space means that a local modification will not cause a change in the overall behaviour of the fuzzy system modelling the process under study. Locality allows the designer to effect local repairs on a fuzzy system which is not functioning correctly for certain situations.

TRACEABILITY

Using only the input data, the designer can trace the partial decisions made by the fuzzy system to obtain a final decision: this can be achieved by listing the fuzzy rules, with their degrees of activation. This characteristic facilitates semantic validation of the fuzzy model, which may be led by experts.

GRANULARITY

The designer is free to choose the size of knowledge granule used, which can be different for each rule of the fuzzy system. Choosing the size of granules used in the fuzzy system effectively sets the size of the cells corresponding to the fuzzy rules in the decision space mesh (Zadeh, 1996). Two approaches are possible:

- The designer can decrease (or increase) the size of the cell by increasing (or decreasing) the number of variables in the premise part of the rule: this enables more detailed modelling of all the factors which determine the differences in the behaviour of the process under study (Figure 4).
- If the behaviour of a rule is considered too rough, it can be locally refined by replacing it with a group of more specialized rules. The mesh is restructured by replacing each fuzzy class qualifying individual rule inputs, with two or more specific classes (Figure 5).

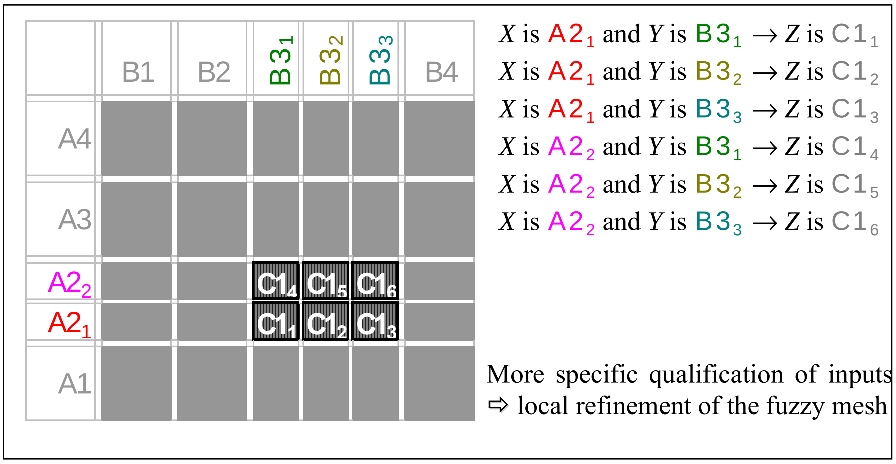


Figure 5 - Granularity of fuzzy modelling

The mesh can be iterated fractally, using successive zooms; it is this property which enables any non-linear function to be approximated to any desired level of precision. Conversely, this explains the frugality of a fuzzy system, where even if there is only a brief or partial description of the decision-making situation, the decision process itself is not seriously degraded.

Unlike the connectionist approach (i.e. formal neuronal networks) which builds black box type models (Hassoun 1995) (Patterson 1996), the automatic induction of fuzzy models proposed by x**tractis**® allows properties of interpretability, traceability, locality and granularity to be maintained. This is the great strength of fuzzy systems as it still allows flexibility in the definition of non-linear models, even in the absence of a priori knowledge.

3. KEY IDEAS ON FUZZY LOGIC (ZALILA 2003)

- Fuzzy logic embraces three co-existing aspects: Imprecision, Uncertainty and Subjectivity.
- In real life, good decisions are regularly taken on the basis of fuzzy information. Fuzzy theory is the formal framework for the modelling and processing of this type of information.
- Fuzzy knowledge contains a great deal of information: trying to transform this fuzzy knowledge into crisp parameters from the beginning of the decision process inevitably introduces bias into the quality of the decision. On the contrary, it is far better to keep this fuzziness during the entire decision process; the fuzzy decision is then only turned into a crisp decision at the very end.
- Fuzzy logic allows the expert knowledge to be modelled in a gradual and nuanced way, and proposes an analogical and approximate mode of reasoning. It allows the design of decision-making support systems which are more effective than classic expert

systems. It also offers an efficient alternative approach for the modelling of complex non-linear processes and phenomena.

- Fuzzy arithmetic allows the modelling and processing of imprecise numeric quantities. It allows the design of predictive models far more faithful to the real world.
- The measure of possibility replaces probability when the decision-maker must evaluate the occurrence of an event on which he/she has little or poor information. This is particularly apparent in problems of multi-criteria decisions or reliability, when the decision-maker has to use information obtained from human sensors (judgement, expert opinions).

In this way, fuzzy theory enhances the palette of modelling, prediction and evaluation methods and improves formalization of complex problems.

4. REFERENCES

Black, M.(1937) Vagueness, an exercise in logical analysis. *Philosophy of science*, *4*, 427-455.

Gigerenzer, G., Todd, P.M., the ABC Research Group (1999) *Simple Heuristics that make us smart*. New York: Oxford University Press.

Hassoun, M.H. (1995) *Fundamentals of Artificial Neural Networks*. Cambridge: MIT Press.

Kosko, B. (1992) Fuzzy systems as universal approximators. *Proceedings of the 1st IEEE International Conference on Fuzzy Systems, FUZZ-IEEE FUZZ92* (pp. 1153-1162), San Diego.

Lukasiewicz, J. (1967) On 3-valued logic. In: *Polish logic 1920-1939*, McCall S., Oxford University Press.

Lukasiewicz, J. (1967) Many-valued systems of propositional logic. In: *Polish logic 1920-1939*, McCall S., Oxford University Press.

Mamdani, E. H. (1974) Application of fuzzy algorithms for control of a simple dynamic plant. *Proceedings of IEEE* (pp. 1585-1588), 121-12.

Mamdani, E.H., Assilian, S. (1975) An experiment in linguistic synthesis with a fuzzy logic controller. *Int. J. Man-Machine Studies*, *7*, 1-13.

Patterson, D.W. (1996) *Artificial Neural Networks. Theory and Applications*. Singapore: Prentice Hall.

Russel, B. (1923) Vagueness. *Australian J. of Philosophy*, *1*.

Wang, L. X., Mendel, J. M. (1992) Fuzzy basis functions, universal approximation and orthogonal least square learning. *IEEE Trans. Neural Networks*, *3*, 807-814.

Zadeh, L.A. (1965) Fuzzy sets. *Information and Control*, *8*, 338-353.

Zadeh, L.A. (1968) Fuzzy algorithms. *Information and Control*, *12*, 94-102.

Zadeh, L.A. (1973), Outline of a new approach to the analysis of complex systems and decision processes. *IEEE Trans. Systems, Man and Cybernetics*, *3*, 28-44.

Zadeh, L.A. (1975), The concept of a linguistic variable and its application to approximate reasoning. *Information Science*, Part I : *8*, 199-249, Part II : *8*, 301-357, Part III : *9*, 43-80.

Zadeh, L.A. (1996), *The Key Roles of Fuzzy Information Granulation in Human Reasoning, Fuzzy Logic and Computing with Words*. Internal Report, Computer Science Division and the Electronics Research Laboratory, Department of EECS, University of California, Berkeley, California, USA, October 15 1996.

Zalila, Z. (1993) *Contribution à une théorie des relations floues d'ordre n*. PhD thesis, D 573, Université de Technologie de Compiègne, France, 474 p.

Zalila, Z. (1993-2013) *Logique floue : concepts et applications*, SY10 coursebook, Université de Technologie de Compiègne, France, 205 p.